Progressive Methods in Multiple Criteria Decision Analysis

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POLYTECH.MONS

Quick decisions are unsafe decisions.

Sophocles (*495 BC - 406 BC*)

A few words on the title

Progressive Methods in Multiple Criteria Decision Analysis

Multiple Criteria Decision Analysis (MCDA):
The aim of an MCDA method is to help you to make a decision related to objects described on more than one preferential dimension.

Multiple Criteria Decision Problem

You need to buy a house and have to chose between one of the following ones:

house	style	price (k€)	area (m^2)
a	**very modern**	200	100
b	old fashioned	160	100
c	classical	**100**	50
d	classical	200	**180**

Progressive Methods: cf. later in this presentation

A few questions beforehand

How can we deal with **impreciseness** in Multiple Criteria Decision Analysis (MCDA)?

How can **missing information** in MCDA be treated?

In decision problems involving **limited economical resources**, how can the decision process be guided?

How can a **prudent** resolution of a decision problem be achieved?

What can be done if the preferential information expressed by a decision maker (DM) is **not in accordance** with the underlying mathematical model?

Context of this work

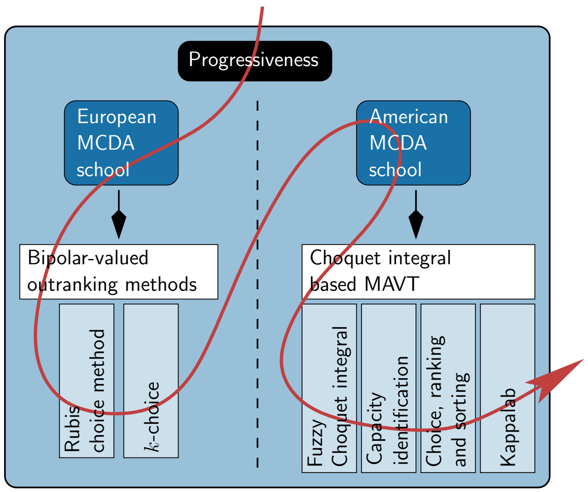


Figure: Context of the thesis and guideline

Structure of the thesis

1. **Progressiveness in MCDA**
   - **Definition**
   - Consequences in both methodological schools
2. **The choice problematique in a bipolar-valued outranking framework**
   - The bipolar-valued outranking relation
   - The RUBIS choice recommendation
   - The *k*-choice problematique
3. **Choquet integral-based MAVT**
   - The Choquet integral and its fuzzy extension
   - The capacity identification problem
   - Common MCDA problems
   - Kappalab

Progressiveness in MCDA

Progressive (*adj.*)

Moving forward, advancing, proceeding in steps, continuing steadily by increments[a].

[a]The American Heritage Dictionary of the English Language, Fourth Edition, © 2000, by Houghton Mifflin Company.

Progressive MCDA method

A *progressive* MCDA *method* is an **iterative** procedure which presents **intermediate recommendations** to the DM which have to be **refined** at a further step of the MCDA.

Progressiveness in MCDA: an example

Choice of a candidate for a position in a company

Step 1 A first filtering on the basis of the résumés and rejection of inappropriate applicants.

Step 2 Telephonic interview with the remaining candidates, and rejection of the unsuitable persons.

Step 3 Psychological tests at the company for a few persons, and rejection of the inapt ones.

Step 4 Invitation of the remaining applicants for an interview with the head of the company, and selection of the *best* candidate.

General scheme

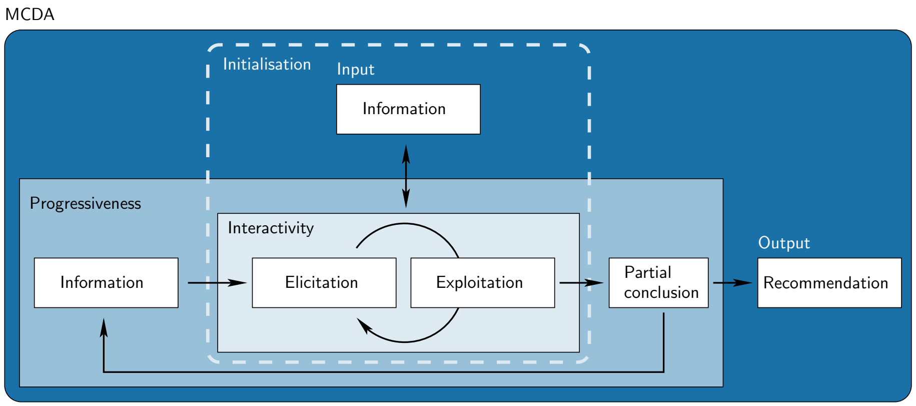


Figure: General scheme of a progressive MCDA

Possible improvements at each loop & objective

- **Enrichment** of the currently available data by supplementary information.
- Resolution of issues linked to **missing** or **incomplete** information.
- Focus on a **subset** of alternatives to refine the recommendation.

↓

Decision maker
who **progressively**
becomes more and more satisfied
with the partial conclusion

Motivations for progressive MCDA

Prudence

At each step:

- The process focusses exclusively on the **available** information at that moment.
- Only **strongly motivated** conclusions are drawn.

Economically constrained problems

In practice:

- Limited **financial** and/or **temporal** resources might not allow to consider all possible points of view for each alternative.
- Certain evaluations might not be available and require to be **postponed** at a later moment in time.

$\Longrightarrow$ *Focus exclusively on the information available at each step.*

Motivations for progressive MCDA

Missing values

Depending on the underlying method:

- **Partial conclusions** should take into account this lack of information.
- The determination of some of this information can be **postponed**.
- No need to make (questionable) **hypotheses** on the missing data (prudence).

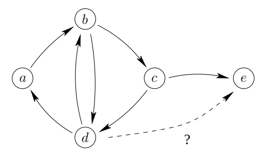
No return policy

- **Irrevocability** of previous partial conclusions.
- **Else:**
  - No guarantee of **convergence** (c.f. choice problematique).
  - Reappraisals **not acceptable** in economically constrained problems.
- Might be **questionable** in some practical situations.

Progressiveness in outranking methods

An **outranking relation** allows 4 types of comparisons of alternatives:

- Preference.
- Indifference.
- Incomparability.
- Indetermination.



Purpose of progressiveness

Resolve incomparability, indifference and/or indetermination situations.

In particular: in the **choice problematique**, progressiveness allows to monotonically reduce the set of potentially best alternatives.

Progressiveness in value function methods

In a **value function** method, the alternatives become comparable via their overall values.
2 types of comparisons of alternatives are therefore possible:

- Strict preference.
- Indifference.

Purpose of progressiveness

Resolve indifference situations and verify *close* alternatives.

In particular:

- In the **ranking** problematique: Focus on (*nearly*) indifferent alternatives to make them *strictly preferentially* comparable.
- In the **choice** problematique: Focus on the first positions of the ranking to determine the single best alternative.

Where are we?

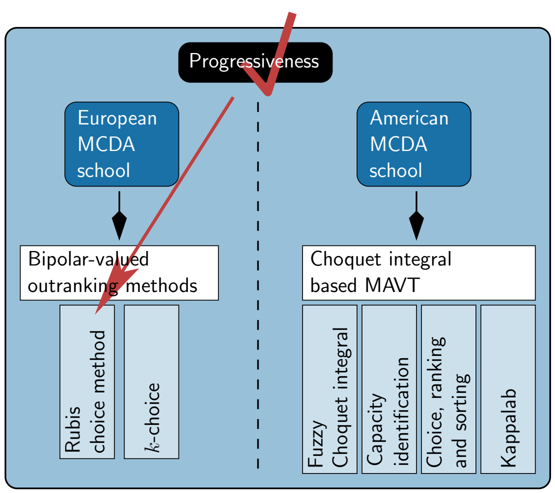


Figure: Context of the thesis

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   - Common MCDA problems
   - Kappalab

The choice problematique

Definition

The **choice problematique** is the category of decision problems consisting of the search for a **single best** alternative (the *choice*).

Definition

The **elimination problematique** is the category of decision problems consisting of the search for a **single worst** alternative.

Plan of attack

We adopt a **progressive** decision analysis process which allows to uncover the choice progressively via **intermediate recommendations**.

Notation

Let X be a finite set of p **alternatives**.

Let N be a finite set of $n > 1$ **criteria**.

Let x and y be two alternatives of X.

Let $g_j(x)$ be the value taken by alternative x on criterion j (also noted x_j later).

Backbone of RUBIS : $\widetilde{S}$

Outranking

Classically, x **outranks** y (xSy) if there is a sufficient majority of criteria which supports an *at least as good as* statement and there is no criterion which raises a veto against it.

Here: We define a **valuation** $\widetilde{S}$ of the outranking relation which expresses the **credibility of the validation** or **non-validation** of the outranking situation.

Backbone of RUBIS : $\widetilde{S}$

Definition

$$\widetilde{S}(x,y) = \min\left\{\sum_{j\in N} w_j \cdot C_j(x,y), -V_1(x,y), \ldots, -V_p(x,y)\right\}$$

$\widetilde{S}$ is defined on a **bipolar-valued credibility scale** $\mathcal{L} = [-1,1]$:

- $\widetilde{S}(x,y) = +1$ means that assertion "xSy" is *clearly validated*.
- $\widetilde{S}(x,y) > 0$ means that assertion "xSy" is *more validated than non-validated*.
- $\widetilde{S}(x,y) = 0$ means that assertion "xSy" is *indetermined*.
- $\widetilde{S}(x,y) < 0$ means that assertion "xSy" is *more non-validated than validated*.
- $\widetilde{S}(x,y) = -1$ means that assertion "xSy" is *clearly non-validated*.

$\widetilde{S}$: an example

alternatives	coherent family of criteria 1	2	3	4	5	$\widetilde{S}$ a	b	c	d	e
a	0.52	0.82	0.07	1.00	0.04	1.0	-0.2	-1.0	0.6	0.4
b	0.96	0.27	0.43	0.83	0.32	0.4	1.0	0.2	0.2	0.4
c	0.85	0.31	0.61	0.41	0.98	0.2	0.4	1.0	0.4	0.6
d	0.30	0.60	0.74	0.02	0.02	-1.0	-1.0	-1.0	1.0	-1.0
e	0.18	0.11	0.23	0.94	0.63	0.2	0.2	-0.4	0.0	1.0

Table: Performance table and bipolar-valued outranking relation

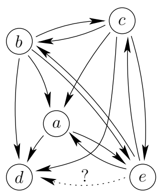


Figure: Associated crisp digraph and indetermined arc

Question:
Which is the best alternative, on the basis of the available information?

5 pragmatic principles for a *choice recommendation* (CR)

$\mathcal{P}_1$: **Non-retainment for well motivated reasons.**
Each non-retained alternative must be considered as worse as at least one alternative of the CR.

$\mathcal{P}_2$: **Minimal size.**
The number of alternatives retained in a CR *should be as small as possible.*

$\mathcal{P}_3$: **Efficient and informative refinement.**
Each step of the progressive decision aiding must deliver an efficient and informative refinement of the previous CR.

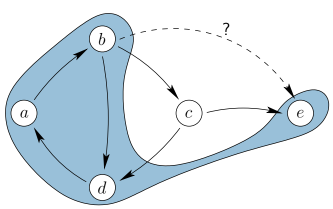
$\mathcal{P}_4$: **Effective recommendation.**
The recommendation should not correspond simultaneously to a choice *and an* elimination *recommendation.*

$\mathcal{P}_5$: **Maximal credibility.**
The CR *must be as credible as possible with respect to the preferential knowledge available in the current step of the decision aiding process.*

Some definitions

Let Y be a non-empty subset of X.

1. Y is said to be **outranking** (resp. **outranked**) in $\widetilde{G}(X,\widetilde{S})$ if and only if $x \notin Y \Rightarrow \exists y \in Y : \widetilde{S}(y,x) > 0$ (resp. $\widetilde{S}(x,y) > 0$).
2. Y is said to be *independent* in $\widetilde{G}(X,\widetilde{S})$ if and only if for all $x \neq y$ in Y we have $\widetilde{S}(x,y) \leqslant 0$.
3. Y is called an **outranking** (resp. **outranked**) **kernel** if and only if it is an outranking (resp. outranked) and independent set.
4. Y is called an **outranking** (resp. **outranked**) **hyperkernel** if and only if it is an outranking (resp. outranked) set which consists of chordless circuits of odd order $p \geq 1$ which are independent of each other.



Translation into formal properties

$\mathcal{P}_1$: **Non-retainment for well motivated reasons.**
A CR *is an outranking set in* $\widetilde{G}(X,\widetilde{S})$.

$\mathcal{P}_2$&$\mathcal{P}_3$: **Minimal size** & **Efficient and informative refinement.**
A CR *is a hyperkernel.*

$\mathcal{P}_4$: **Effective recommendation.**
A CR *is a strict set in* $\widetilde{G}(X,\widetilde{S})$.

$\mathcal{P}_5$: **Maximal credibility.**
A CR *has maximal determinateness.*

Theorem
Any outranking digraph contains at least one outranking and one outranked hyperkernel.

Rubis choice recommendation

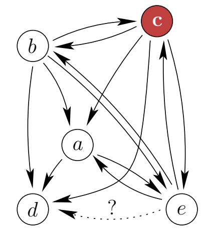
A Rubis **choice recommendation** is a recommendation which verifies the 5 pragmatic principles.

By construction, in $\widetilde{G}(X, \widetilde{S})$, a Rubis choice recommendation is given by a **maximally determined strict outranking hyperkernel**.

Different scenarios:

- The Rubis choice recommendation Y contains **more than one** element:
  Continue the progressive analysis on Y.
- No **strict** choice recommendation exists:
  Continue the progressive analysis on X and try to determine a less *symmetrical* situation.
- **Multiple** Rubis choice recommendations exist:
  To be prudent, continue the progressive analysis on the union of all the recommendations.

An example: continued



$\widetilde{Y}$	a	b	**c**	d	e	$D(\widetilde{Y})$	
{**c**}	-0.2	-0.4	**0.4**	-0.4	-0.4	**0.36**	RCR
{b}	-0.2	**0.2**	-0.2	-0.2	-0.2	**0.20**	-

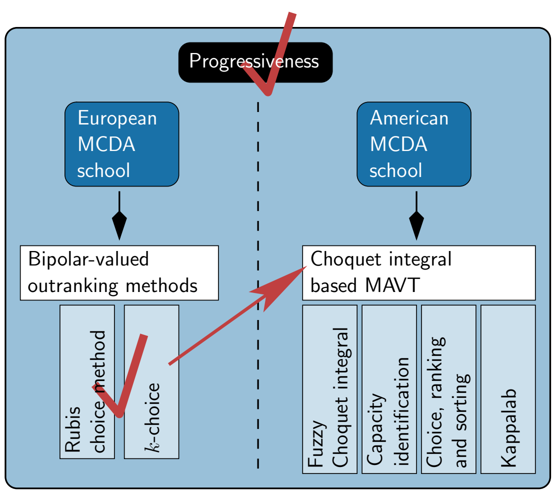
Also in the manuscript

Algorithmic issues and experimental study on the Rubis method.

Introduction of the k-choice problematique.

Extension of Rubis to the k-choice problematique.

Where are we?



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   - **The capacity identification problem**
   - Common MCDA problems
   - Kappalab

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Multiattribute Value Theory (MAVT)

Objective

Model the preferences of the DM, represented by the relation $\succeq$ on X by means of an overall value function $U : X \to \mathbb{R}$:

$$x \succeq y \iff U(x) \geq U(y).$$

Model often used: *additive value function model*

Here:

Transitive decomposable model

$$U(x) := F(u_1(x_1), \ldots, u_n(x_n)),$$

where the functions $u_i : X_i \to \mathbb{R}$ are called the *marginal value functions* and $F : \mathbb{R}^n \to \mathbb{R}$, non-decreasing in its arguments, is called the *aggregation function*.

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Towards Choquet integral-based MAVT

When *mutual preferential independence* among criteria can be assumed, F is often taken as a **weighted sum**.

Here: to take interaction phenomena among criteria into account, the weight vector of the weighted sum is replaced by a **capacity** on N.

A natural extension of the weighted sum is then the **Choquet integral**.

In such a context, it is necessary that the marginal value functions are **commensurable**.

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Towards Choquet integral-based MAVT

Problem: determination of the parameters of the *capacity* (at most $2^n - 2$)!

It is advisable to determine the capacity from some **learning data** (called the *initial preferences* of the DM).

Once the parameters of the capacity have been determined, the overal value of each alternative can be calculated.

The alternatives become *comparable* and the MCDA problem can be solved.

The Choquet integral

Let $x : N \to \mathbb{R}$ represented by the vector $(x_1, \dots, x_n)$

Definition

The **Choquet integral** of x w.r.t to a capacity μ on N is defined by:

$$\mathcal{C}_\mu(\mathbf{x}) := \sum_{i=1}^{n} x_{(i)}[\mu(A_{(i)}) - \mu(A_{(i+1)})]$$

where μ represents a capacity on N,
that is a monotone set function $\mu : 2^N \to [0,1]$ fulfilling $\mu(\emptyset) = 0$ and $\mu(N) = 1$.
The parentheses used for indexes represent a permutation on N such that

$$x_{(1)} \leq \dots \leq x_{(n)},$$

and $A_{(i)}$ represents the subset $\{(i), \dots, (n)\}$.

Note: In the thesis we extend the Choquet integral to aggregate **fuzzy partial evaluations**.

The capacity identification problem: some initial preferences

- a partial weak order $\succeq_O$ over $O \subseteq X$ (**ranking** of the available objects);
- a partial weak order $\succeq_N$ over N (ranking of the **importance** of the criteria);
- intuitions about the type and the **magnitude** of the interaction between some criteria;
- etc.

Note: Each of them can be translated in a **linear constraint**.

The capacity identification problem: mathematical formulation

min or max f

subject to

$$\begin{cases} \mu(S \cup i) - \mu(S) \geq 0, \forall i \in N, \forall S \subseteq N \setminus i, \\ \mu(N) = 1, \\ C_\mu(u(x)) - C_\mu(u(x')) \geq \delta_C, \\ \vdots \\ \phi_\mu(i) - \phi_\mu(j) \geq \delta_{Sh}, \\ \vdots \\ I_\mu(ij) - I_\mu(kl) \geq \delta_I, \\ I_\mu(kl) \geq 0 \\ \vdots \end{cases}$$

where f is an objective function that distinguishes the different identification methods.

The capacity identification problem: solution

Solution: general capacity defined by $2^n - 1$ coefficients.

3 main reasons for an **infeasible** problem:

- Preferential information violates **natural MCDA axioms**.
- Number of **parameters** of the model is too small.
- Preferential information violates axioms specific to the **Choquet integral** model.

The capacity identification problem: our proposal

$$\min \mathbf{f_{GLS}(m_\mu, y)} := \sum_{\mathbf{x \in O}} \left[\mathbf{C_{m_\mu}(u(x)) - y(x)}\right]^2$$

$$\text{subject to} \begin{cases} \sum_{\substack{T \subseteq S \\ t \leq k-1}} m_\mu(T \cup i) \geq 0, \forall i \in N, \forall S \subseteq N \setminus i, \\ \sum_{\substack{T \subseteq N \\ 0 < t \leq k}} m_\mu(T) = 1, \\ \mathbf{y(x) - y(x') \geq \delta_y}, \\ \vdots \\ \phi_{m_\mu}(i) - \phi_{m_\mu}(j) \geq \delta_{Sh}, \\ \vdots \\ I_{m_\mu}(ij) - I_{m_\mu}(kl) \geq \delta_I, \\ I_{m_\mu}(kl) \geq 0 \\ \vdots \end{cases}$$

where $y = \{y(x)\}_{x \in O}$ are variables representing overall **unknown** evaluations of the objects that must verify the weak order imposed by the DM.

The capacity identification problem: our proposal

2 possibilities for the output

1. **If** $f_{GLS}(m_\mu, y) = 0$
   then $y(x) = C_{m_\mu}(u(x)), \forall x \in O$.

   Difficulty: Not necessarily a unique solution!

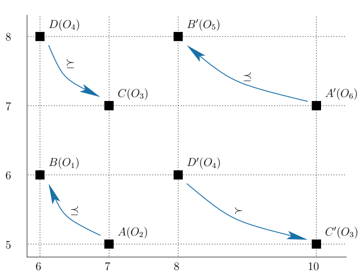
   Suggestion: Use another identification method.

2. **If** $f_{GLS}(m_\mu, y) > 0$
   then the $\{y(x)\}_{x \in O}$ do not match the $\{C_{m_\mu}(u(x))\}_{x \in O}$.
   - Either the DM's weak order on O is violated,
   - Or δ_y is not respected.

Solving common MCDA problems

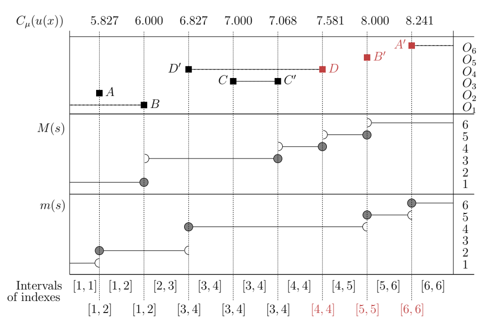
In the thesis, we show how the **choice**, **ranking** and **sorting** problematiques can be solved in the crisp and the fuzzy case.

Example: a sorting problem



Preference relation revealing **comonotonic contradictory tradeoffs**

∄ Choquet integral!



3 alternatives (A', B' and D') assigned precisely

5 alternatives assigned ambiguously

Progressiveness in Choquet integral-based MAVT

Fuzzy case:

Choice problematique:	Narrow the fuzzy numbers to obtain a smaller **core**.
Ranking problematique:	Narrow the fuzzy numbers to obtain a less **weak** order.

Crisp case:

Choice problematique:	Focus on the **first positions** of the weak order to check if the order is precisely defined.
Ranking problematique:	Try to make a **less weak** order.
Sorting problematique:	Try to make a **less ambiguous** classification.

Also in the manuscript

Extension of the Choquet integral to fuzzy numbers.

Unified presentation of different capacity identification methods.

The resolution of common MCDA problems in the fuzzy and the crisp case.

Detailed presentation of the use of Kappalab in an interactive process.

Where are we?

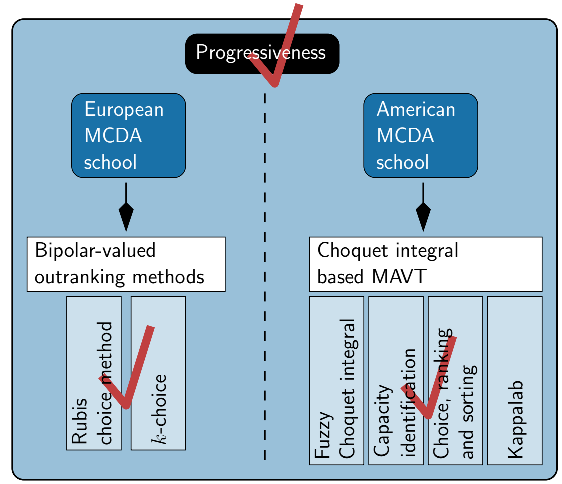


Figure: Context of the thesis

Concluding remarks

Progressiveness can be an interesting framework to guide an MCDA process

Our research is situated in the two main methodological schools for MCDA

The **DM** has a central position in the decision process and in our considerations

Many answers, but still **open questions**:

Perspectives

Availability of the DM in progressiveness: how could we obtain the final recommendation in the next step?

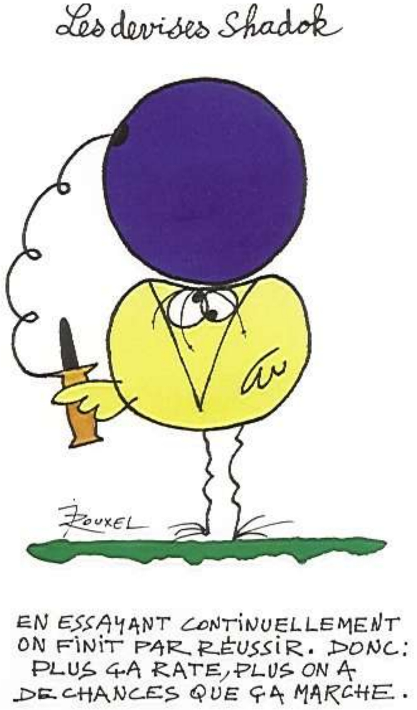
Give a **recipe** on how to use the RUBIS method in a real-world situation.

Further research on the *k*-choice problematique.

The **properties** and **structure** of a bipolar-valued outranking relation.

Robustness, **quality**, **reliability** and **degree of indeterminacy**.

Capacity identification: determine a method which represents the **weak** order on the alternatives as accurately as possible.



By trying day in and day out,
one always succeeds.
Thus: the more things fail,
the more they may work out.

Thank you for your attention.