Software

Multiple Criteria Sorting: TOMASO
A Solution in the Presence of Interacting Points of View

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Abstract

This short article briefly presents the main features of the multiple criteria sorting tool TOMASO (**T**echnique for **O**rdinal **M**ulti-**A**ttribute **S**orting and **O**rdering) and its implementation. Its main particularities are the possibility to consider interacting points of view and the use of the Choquet integral as a discriminant function. The capacities are learnt through the use of protoypes, which are well known alternatives for the Decision Maker.

1. Introduction

Let A be a set of a potential alternatives and $F=\{g_1,\ldots,g_n\}$ be a set of ordinal points of view. Each alternative is evaluated on each of the points of view. For each index of point of view $j\in J=\{1,\ldots,n\}$, this evaluation is done according to a s_j-point ordinal performance scale represented by a totally ordered set $X_j:=\{g_1^j \prec \ldots \prec g_{s_j}^j\}$. Therefore, an alternative $x\in A$ can be identified with its corresponding profile $(x_1,\ldots,x_n)\in\prod_{j=1}^{n} X_j =: X$, where for any $j\in J$, x_j is the partial evaluation of *x* on point of view *j*. Let us now suppose that the Decision Maker (DM) would like to assign the alternatives of A to m increasingly ordered classes $\{Cl_t\}_{t=1}^m$ (which means that for any $r>s\in\{1,\ldots,m\}$ the elements of Cl_r are considered as better than the elements of Cl_S). The objective is therefore to partition A into the classes $\{Cl_t\}_{t=1}^m$.

Most of the classical Multiple Criteria Decision Aiding methods use the classical weighted sum as an aggregator. In order to allow interaction among the points of view, we use the Choquet integral [Cho] as a discriminant function. For an alternative a and its corresponding profile $(x_1,\ldots,x_n)$ it is defined by $C_v(x):=\sum_{i=1}^{n} x_{(i)}\left[v(A_{(i)})-v(A_{(i+1)})\right]$ where v is a fuzzy measure on J, the parentheses used for indices represent a permutation on J such that $x_{(1)}\le\ldots\le x_{(n)}$ and $A_{(i)}$ stands for the subset $\{(i),\ldots,(n)\}$.

The use of the Choquet integral as an aggregator allows to calculate numerical upper and lower boundaries of the classes. Nevertheless, the DM is not asked to provide any technical information on weights (capacities of the Choquet integral) or thresholds. He should only provide a set of prototypes $P\subseteq A$. A prototype is a well-known alternative for the DM. He must be aware of the global quality of each prototype in order to assign each of them to one and only one of the predefined classes. The prototypes can be fictitious elements which are not necessarily among the analysed alternatives. Nevertheless, they should be potentially existing alternatives because information will be extracted from their assignments to the classes.

In the following section we briefly present each of the steps of the TOMASO procedure. The interested reader should refer to [Mar] or [Mey] for details.

2. General ideas on the method

The different stages of the TOMASO technique are listed hereafter :

1. Modification of the original ordinal evaluations into normalised scores ;
2. Definition of the set of prototypes, and assignment of the prototypes to the predefined classes ;
3. Assessment of the capacities of the Choquet integral by solving a linear or a quadratic program ;
4. Calculation of the numerical boundaries of the classes ;
5. Assignment of the prototypes and the remaining alternatives of A to the classes ;
6. Analysis of the results.

In the first step, a scoring approach is used to allow us to work on the same scale for each point of view. Such scores, whose definition might vary from an application to another, should have a precise meaning for the decision maker.

Two natural approaches can be considered: either the score of each alternative is built on the basis of all the alternatives in A or this score is constructed in a context-free manner, that is, independently of the other alternatives. The decision maker must be aware that the final results may significantly differ according to the considered approach. Therefore, a prior analysis of the problem is recommended to choose the scores appropriately.

In the first approach, one possible way to build the scores is to consider comparisons of the alternatives on each of the points of view. We define the j th partial net score of alternative x in A along point of view j, as the number of times that x is preferred to any other alternative of A minus the number of times that any other alternative of A is preferred to x for point of view j. We furthermore normalize these scores so that they range in the unit interval. Clearly, this normalized score is not a utility, and should not be considered as such. Indeed, observing an extreme value (close to 0 or 1) means that x is rather ***atypical*** compared to the other alternatives along point of view j. Thus, the resulting evaluations strongly depend on the alternatives which have been chosen to build A.

Consider now the second approach, that is, where the score of each alternative does not depend on the other alternatives in A. In this case, we suggest the decision maker provides the score functions as utility functions. Alternatively, we can approximate these utility functions by linear ones. These functions do not necessarily represent a real utility and probably do not correspond to the utility the decision maker has in mind. We therefore continue to call it a score. Notice that the case study we present in the next section is treated by means of the scores of the first type, i.e., based on the comparison of alternatives.

The second stage of TOMASO consists in defining the prototypes by assigning elements of A to the classes. Each class should be « described » by at least one element. The assessment of the fuzzy measure of the Choquet integral is then done by « learning » from the information provided by the prototypes.

In case the prototypes don’t violate the axioms for the existence of a Choquet integral as a discriminant function [Wak], a linear constraint satisfaction problem is solved (TOMASO 1). The unknowns are the coefficients of the fuzzy measure. The resulting capacities are then used to define the numerical limits of each of the clearly separated classes (maximum and minimum).

In case the prototypes violate for example the triple cancellation axiom [Wak], the Choquet integral cannot be used as a discriminant function. In that case, we solve a quadratic problem where the unknowns are the capacities of the Choquet integral and a global evaluation (score) for each alternative which respects the sorting imposed by the DM on the prototypes (TOMASO 2). The goal is to minimise the distance between the values of the Choquet integral and the global evaluations. The resulting capacities are then used to define the numerical boundaries of the classes, which are not necessarily well separated.

In the first ideal scenario, each prototype is correctly assigned to the classes, with respect to the DM’s classification. The Choquet integral of the remaining alternatives is then calculated, and each of them is assigned to a single class or the union of two classes.

In the second case, the prototypes are not necessarily correctly assigned to a single class. It may happen that the classes overlap or that the prototypes are not classified according to the DM’s classification. Similarly to the ideal scenario, the Choquet integral of the remaining alternatives is then calculated. Ambiguous assignments to more than one class can occur. It is possible to force each of the alternatives of A to belong to a single class after the assignment. This is done by a k-nearest neighbour approach for the classification.

After the assignments of the prototypes and the remaining alternatives, it can be interesting to analyse the behaviour of the fuzzy measure. This is done through two indexes, namely the Shapley index for the importance of each point of view, and the interaction index. But at this point, the user must be aware of an important fact : any information which is extracted from the assignment of the prototypes depends on the definition of the set A of potential alternatives and the subset P. The importance and interaction indexes are therefore only valid for the given problem, and should not be taken out of their context.

Let us finally show the use of the method on a classification problem in the next section.

3. Application

The TOMASO method is implemented in a freeware which can be obtained from the authors. A tutorial regarding the method can also be found there. As the research on this multiple criteria sorting procedure is still in progress, the software is regularly updated and improved. Nevertheless we show how the method behaves on a small example which is presented in further details in [Mar].

Consider the classical example of 27 different students evaluated on 3 courses (Mathematics, Physics and Literature) on a qualitative ordinal scale with 3 levels : bad (B) < medium (M) < good (G). Each student has to be assigned to one of the following 3 classes : bad < medium < good. A teacher has chosen to assign the follwing students as prototypes to the 3 classes:

Good	G,GM	G,G,M	G,M,G	G,G,G				
Med.	G,M,B	G,G,B						
Bad	G,B,B	B,M,B	B,G,B	G,B,M	B,G,M	G,B,G	B,M,G	B,G,G

Table 1 : The prototypes

The objective is to assign the remaining 13 students to the classes, according to the preferences of the DM, expressed by the prototypes. A solution exists for a 2-additive [Mey] fuzzy measure. The importance and interaction indexes are given in the following table:

Importance indexes			Interaction Indexes		
Mat	**Phy**	**Lit**	**Mat/Phy**	**Mat/Lit**	**Phy/Lit**
0.5	0.3	0.2	0.2	0	0.4

Table 2 : Importance and Interaction indexes

The remaining students are assigned to the classes as given in the following table :

od	M,G,G								
d.	M,M,G	M,G,M	M,M,M						
d	B,B,B	M,B,B	M,M,B	M,G,B	B,B,M	M,B,M	B,M,M	M,B,G	B,B,G

Table 3 : Assignments of the remaining students

As we have already mentionned earlier, the importance and interaction indexes only apply to this particular example. If the teacher decides to change his prototypes or the set *A* of the 27 students (by reducing it for example), the model should be recalculated.

4. Conclusions

This article has presented a few ideas on the TOMASO method, and its application to a small example. Its main advantage is its ability to cope with interacting points of view. Furthermore, the DM does not have to provide difficult technical information for the calculation of the model. Some work is currently done on the building of models in case some information is known on the interaction and / or importance indexes (ranking, approximative value,…). Besides, the software is constantly improved, and new graphical tools are being developped to provide easier and more readable information for the DM.

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Generic Multi-Attribute Analysis System

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The **G**eneric **M**ulti-**A**ttribute **A**nalysis (GMAA) System is a *Decision Support System* (DSS) based on an additive multi-attribute utility model that accounts for incomplete information concerning the inputs and is intended to allay many of the operational difficulties involved in the *Decision Analysis* cycle.

The user can interactively create or delete nodes and branches to build or modify an objectives hierarchy. Alternatives and their consequences, in terms of the attributes associated with the lowest-level objectives, can be easily entered by hand or loaded from file. The system admits uncertainty about consequences.

The system also admits incomplete information about the DM’s preferences through value intervals as responses to the probability questions the DM is asked, which leads to classes of utility functions and weight intervals. This is less demanding for a single DM and also makes the system suitable for group decision support.

The different alternatives under consideration can be evaluated by means of an additive multiattribute utility function. The additive model is used to assess, on the one hand, average overall utilities, on which the ranking of alternatives is based and, on the other, minimum and maximum overall utilities, which give further insight into