Use of an ordinal sorting technique (TOMASO) in stock selection

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Abstract. This paper briefly presents the evolutions of the ordered sorting procedure TOMASO based on the Choquet integral as a discriminant function. The method uses information provided by the Decision Maker (DM) in terms of a set of prototypes (alternatives well-known to the DM). This questioning is restricted to his expertise domain and not to technical parameters linked to the method. The method works in two steps. First of all, the ordinal data is transformed into partial net scores, where each alternative is compared to all the other ones for each point of view. Then, the Choquet integral is used to aggregate these partial net scores. The values of the capacity of the aggregator are learnt from the reference set of prototypes. This note finally shows that it is possible to use TOMASO in a stock selection procedure.

Keywords: TOMASO, Stock selection, Choquet integral.

1 Introduction

In this short paper, we briefly describe the main features of the sorting procedure for ordinal data in a very general case, where the points of view can have interactions. Its name, TOMASO stands for **T**ool for **O**rdinal **M**ulti**A**ttribute **S**orting and **O**rdering. The first version of this method has been described in [Marichal and Roubens, 2001] and [Roubens, 2001].

Later, in [Marichal *et al.*, 2005] the authors present further evolutions to the first ideas, and describe a software which is directly inspired from the sorting procedure. The present paper intuitively presents evolutions to these original methods, namely the solving of a quadratic program to determine the model. For a detailed description of the latest advances the interested reader should refer to [Meyer and Roubens, 2005].

Three important features characterise this method. First of all, the possibility to treat purely ordinal data. Secondly, the use of a Choquet integral [Choquet, 1953] as a discriminant function. And finally, the way the values of the capacity ("weights") of the aggregator are learnt from a reference set of alternatives called prototypes. These three key features allow to treat a quite large set of problems. In particular, the learning feature of the method is interesting as it allows to ask the Decision Maker (DM) a minimal set of technical details. In order to allow a more effective and objective analysis of the problem, we think that it is useful to have a permanent interaction with the DM. But this questioning should mainly be restricted to his expertise domain and not to technical parameters of the method. The use of the prototypes fits to this philosophy.

The article is organised as follows. First of all, general concepts are introduced in Section 2 and an intuitive description of the method is provided. Then, in Section 3 we show that a stock selection procedure can be aided by TOMASO. Finally, in Section 4 we draw some conclusions, and discuss further improvements.

2 Methodological considerations

The objective of this Section is to present a short and intuitive description of the TOMASO method. The interested reader can refer to [Marichal *et al.*, 2005] and [Meyer and Roubens, 2005] for a description of the specificities of this ordinal sorting procedure. We merely present the general ideas of the method in view of the application presented in Section 3.

2.1 Preliminary reflections

Let A be a set of q potential alternatives which are to be assigned to disjoint ordered classes. Let $F = \{g_1, \ldots, g_n\}$ be a set of points of view. For each index of point of view $j \in \mathcal{J} = \{1, \ldots, n\}$, the alternatives are evaluated according to a s_j-point ordinal performance scale represented by a totally ordered set $X_j := \{h_1^j \prec_j \ldots \prec_j h_{s_j}^j\}$. An alternative $x \in A$ can be identified with its corresponding profile

$$(x_1, \ldots, x_n) \in \prod_{j=1}^{n} X_j =: X,$$

where for any $j \in \mathcal{J}$, x_j is the partial evaluation of x on point of view j.

Consider a partition of $X := \Pi_{i=1}^{n} X_j$ into m nonempty increasingly ordered classes $\{Cl_t\}_{t=1}^{m}$. This means that for any $r, s \in \{1, \ldots, m\}$, with $r > s$, the elements of Cl_r are considered as better than the elements of Cl_s. The sorting problem we are dealing with consists in assigning the alternatives of A to the classes $\{Cl_t\}_{t=1}^{m}$.

Roubens [Roubens, 2001] justifies how an n-place Choquet integral as a discriminant function and normalised scores as criteria function can be used to solve this problem.

2.2 Building the evaluations

Let us explain how the original ordinal scales are treated in view of an aggregation by means of a Choquet integral. Two natural approaches can be considered: either the score of each alternative is built on the basis of all the alternatives in A or this score is constructed in a context-free manner, that is, independently of the other alternatives. The DM must be aware that the final results may significantly differ according to the considered approach. Therefore, a prior analysis and a good understanding of the problem is recommended to choose the scores appropriately.

In the first approach, one possible way to build the scores is to consider comparisons of the alternatives on each of the points of view. We consider $S_j(x)$, the jth partial net score of alternative $x \in A$ along point of view $j \in \mathcal{J}$, as the number of times that x is preferred to any other alternative of A minus the number of times that any other alternative of A is preferred to x for point of view j. We furthermore normalise these scores so that they range in the unit interval, i.e.,

$$S_j^N(x) := \frac{S_j(x) + (q-1)}{2(q-1)} \in [0,1] \qquad (j \in \mathcal{J}),$$

where $q = |A|$. Clearly, this normalised score is not a utility, and should not be considered as such. Indeed, observing an extreme value (close to 0 or 1) means that x is rather "atypical" compared to the other alternatives along point of view j. Thus, the resulting evaluations strongly depend on the alternatives which have been chosen to build A.

Consider now the second approach, that is, where the score of each alternative does not depend on the other alternatives in A. In this case, we suggest that the DM provides the score functions as utility functions. Alternatively, we can approximate these utility functions by the following linear formula:

$$S_j^N(x) := \frac{\mathrm{ord}_j(x) - 1}{s_j - 1} \in [0,1] \qquad (j \in \mathcal{J}),$$

where $\mathrm{ord}_j : A \to \{1, \ldots, s_j\}$ is a mapping defined by $\mathrm{ord}_j(x) = r$ if and only if $x_j = h_r^j$. In this latter case, S_j^N does not necessarily represent a real utility and probably does not correspond to the utility the decision maker has in mind. We therefore continue to call it a score.

2.3 Moving towards the aggregation

The next step of the method is the aggregation of the normalised partial net scores of a given alternative x by a Choquet integral [Choquet, 1953]. The advantage of this aggregator is mainly that it allows to deal with interacting (depending) points of view. According to the general definition of the

Choquet integral, we have in this particular case:

$$\mathcal{C}_v(S^N(x)) := \sum_{j=1}^{n} S^N_{(j)}(x)[v(A_{(j)}) - v(A_{(j+1)})]$$

where v is a fuzzy measure on $\mathcal{J}$; that is a monotone set function $v : 2^{\mathcal{J}} \to [0,1]$ fulfilling $v(\emptyset) = 0$ and $v(\mathcal{J}) = 1$. The parentheses used for indexes stand for a permutation on $\mathcal{J}$ such that

$$S^N_{(1)}(x) \leq \ldots \leq S^N_{(n)}(x),$$

and for any $j \in \mathcal{J}$, $A_{(j)}$ represents the subset $\{(j), \ldots, (n)\}$.

This fuzzy measure (or capacity in this context of the Choquet integral) merely expresses the importance of each subset of points of view. If points of view cannot be considered as independent, the importance of subsets $v(S)$, $S \subseteq \mathcal{J}$ has to be taken into account. The Choquet aggregator presents usual desirable properties. It is continuous, non-decreasing, located between min and max and its characterisation [Marichal, 2000] clearly justifies the way the partial scores have been aggregated.

One can easily understand that it is impossible to ask the DM to give values for the $2^n - 2$ free parameters of the capacity v. Firstly because of the high number of values he should provide, and secondly due to the unclear meaning of the values of the capacity. The information required to build a classification model is therefore obtained by asking the DM to provide a set of prototypes $P \subseteq A$ and their assignments to the given classes; that is a partition of P into prototypic classes $\{P_t\}_{t=1}^m$ where $P_t := P \cap Cl_t$ for $t \in \{1, \ldots, m\}$. The prototypes are well-known alternatives to the DM. As an expert of the analysed field, he has an a priori knowledge which allows him to assign them to the predefined classes. Clearly, this global evaluation depends on the remaining alternatives of $A \setminus P$. Details on how the values of the capacity are derived from this information can be found in [Marichal *et al.*, 2005].

For the purpose of this paper, let us intuitively explain the main steps of this determination. First of all, two possibilities appear: either the assignment of the prototypes is compatible with a Choquet integral as a discriminant function, or some prototypic elements violate the axioms that are imposed to produce a discriminant function of Choquet type ([Marichal, 2000] [Wakker, 1989]), in particular the triple cancellation axiom.

2.4 The ideal scenario

In the first case, some separation conditions (the Choquet integral should strictly separate the classes by $\varepsilon > 0$) put together with the monotonicity constraints on the capacity form a linear program [Marichal and Roubens, 2001] whose unknowns are the values of the capacity, $v(S)$, $S \subset \mathcal{J}$. Furthermore, the non-negative variable ε is to be maximised in order to deliver well separated classes. It is important to recall that the only information which is provided by the DM is the assignment of some prototypes to the predefined classes.

We use the principle of parsimony for the resolution of this problem. This means that we search for a k-additive capacity ([Grabisch, 1997]) v^* which is a solution of the linear program with k as low as possible. The boundaries of the classes are then determined by considering the lowest and the highest values of the Choquet integral of the prototypes in each class. From this point on, the Choquet integral of any alternative x of A can be calculated. This allows to assign x to one of the predefined classes (or to an interval of classes).

2.5 The general scenario

As already stated, it may happen that the linear program described previously has no solution if some conditions are not fulfilled. In such a case, and in order to provide a solution, we suggest to find a capacity which respects as well as possible the classification of the prototypes given by the DM. This is done through the resolution of a quadratic program where we try to minimise the gap between the assignment imposed by the DM and the classification resulting from the aggregation. Intuitively, for a given alternative $x \in P$, its Choquet integral $\mathcal{C}_v(S^N(x))$ should be as close as possible to an unknown global evaluation $y(x)$, which respects the classification imposed by the DM on the prototypes. The flexibility lies in the fact that the Choquet integral of the alternatives is not constrained by monotonicity conditions which might violate for example the triple cancellation axiom.

In this case the positive variable ε which ensures (if possible) a strict separation of the classes, plays the role of a parameter, which needs to be fixed by the DM. A correct choice of ε remains one of the main challenges of our future research.

The boundaries of the classes are determined similarly as in the ideal scenario described previously. The difference lies merely in their final structure. Two or more classes may overlap, and the classification of the alternatives of A becomes an assignment to intervals of classes. Therefore, the choice of a good compromise between the complexity of the model and the proper classification of the prototypes is not to be underestimated. One should favour a complex model (high value of k) with less overlapping classes compared to a simple model (low k) with large overlappings.

The assignment of a prototype a to the intervals of classes leads now to two scenarios:

- a is assigned to a single class (interval of length 0) which corresponds to the original class decided by the DM
- a is assigned to an interval of classes and the original class decided by the DM belongs to this interval.

In the following section we show how TOMASO can be used in the context of financial stock selection.

3 Application to stock selection

In the framework of portfolio management, one important task is the selection of the appropriate stock. These stocks can be described by a quite large number of characteristics, such as fundamental figures, financial ratios, price, signals from technical analysis or recommendations from analysts. These financial analysts' recommendations are generally based on the other cited figures and information directly inspired from the market. They give a hint on the quality of a stock and indicate if it should be bought, sold or kept in an existing portfolio. The objective of the small application we present in this Section is to mimic an expert's decisions on a set of stocks. We first present the data and the general objectives. Then we detail the results of TOMASO and draw conclusions.

3.1 Preliminary considerations

Our work with financial experts has clearly shown that the analysis of stocks in view of a recommendation is a multiple criteria decision problem. Furthermore, it can be considered as a ordered sorting problem. The starting point of this analysis is the desire to see if the decisions of a financial analyst can be modelled. The goal is not to replace him by some type of algorithm, but rather to aid him in his decisions.

The matter which interests us here is a set of 22 stocks from the banking sector. Each of these 22 stocks is described on 9 ordinal variables used by the financial expert to decide on its quality. The scales on each of the variables have 5 levels going from very bad to very good. The variables are listed hereafter:

- Capital Adequacy Ratio
- P/NAVPS estimate
- Price Earnings Ratio estimate
- Return on equity estimate
- Cost income estimate
- Analysts' consensus
- Volatility
- Price Earnings Ratio current year
- Earnings Estimate 4 weeks change estimate

The 22 stocks have been evaluated twice on these criteria. Firstly in October 2000 and secondly in April 2001, 6 months later. In 2000, the analyst has also given a global evaluation for each of the 22 stocks. These recommendations are representable by one of the following three classes: buy, hold, sell. A natural order on these classes is given by the following ordered set: Rec = {buy $\succ$ hold $\succ$ sell}. The objective of this analysis is to assign the 22 stocks of 2001 to one of these 3 classes by taking into account the recommendations of 2000. In some way, one could say that the goal is to mimic the financial analyst.

These considerations lead us to define the set A of alternatives as the set of 44 stocks (22 stocks evaluated in 2000 and 2001). The prototypes are then the alternatives which obtained a global recommendation in October 2000.

Let us note that the sole information provided by the analyst is knowledge from his domain of expertise. No technical parameters are asked.

3.2 TOMASO's models

In order to determine the model (or the capacity which will provide the Choquet integral and the boundaries of the classes), we decide to build the scores on basis of the set A. We consider that the analyst has taken his decisions by restricting himself to this set.

This problem can be solved by using the linear program, and k-additive solutions exist for $k \geq 3$. They are very similar in terms of the assignments of the 22 stocks evaluated in 2001. As even for $k = 3$ the solution is quite complex (hard to determine interactions among criteria, no easy reading of the capacity), we decide to analyse hereafter the solution for $k = 9$. But as already stated, the results are very comparable to the simplest model for $k = 3$.

Each of the 22 alternatives of 2001 has been assigned either to a single class or to a union of two neighbour classes. Unfortunately, it was impossible to get the analyst's classification on these 22 alternatives to validate our results. We therefore suggest to test empirically TOMASO's assignments by making a small portfolio management simulation. This should not be considered as an ultimate proof, but merely as a small indication on the quality of the classification.

3.3 Portfolio simulation

The simulation we are performing here is very naive. We suppose that on March 31, 2001 a portfolio exists with the 22 stocks equally present. This means that each stock represents 1/22 of the entire sum invested in the portfolio. On April 1, 2001 the portfolio is revised with the following strategy:

- Remove all stocks from the portfolio which have been assigned to the sell class by TOMASO
- Buy one more portion of each stock assigned to the buy class by TOMASO
- Leave the remaining stocks unchanged

Figure 1 shows two scenarios for the 22 stocks. The first one represents the evolution of the revised portfolio, taking into account the information provided by TOMASO. The second one is the evolution of the original portfolio

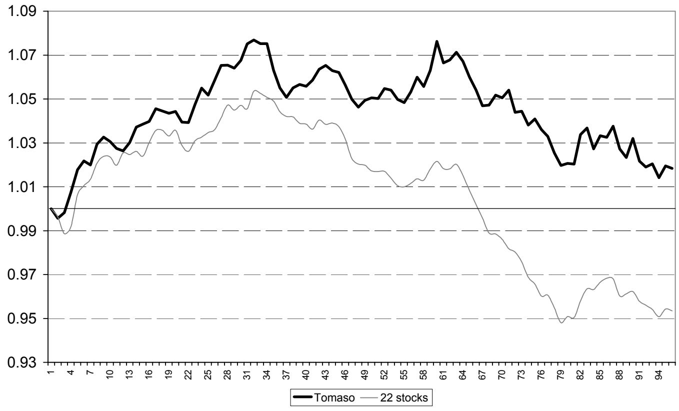

Fig. 1. Evolution of the value of both portfolios

of 22 stocks (equally represented) without any change in the structure, starting on April 1, 2001. The abscissa of the graph represents work days. We can see that the information provided by TOMASO is beneficial to the revised portfolio. It's value is permanently above that of the unrevised one. A more detailed analysis of the different recommendations of TOMASO shows that the buy recommendations clearly outperform the original portfolio on a period of 3 months starting in April 2001. The sell recommendations of TOMASO are in general performing worse than the original portfolio. As already stated earlier, this small simulation is not a proof of the ability of TOMASO to mimic the financial analyst.

4 Concluding remarks

We have briefly presented a procedure for ordinal sorting in the presence of interacting points of view. Details on the procedures can be found in [Marichal *et al.*, 2005] and [Meyer and Roubens, 2005]. Future work will concern the simplification of the software in order to make it even more user-friendly. Furthermore, the automatic determination of the separation of the classes in case of the quadratic program will also be one of our major concerns. The implementation of other indexes (veto, favour, ...) is also planned.

From a practical point of view, we have shown how the method can be applied in a stock selection procedure. The sole information which is used comes from the expertise of the financial analyst. He does not have to deal with technical parameters of the method. The results are promising, but must be considered with much care.

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