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# Association rule interestingness measures: experimental and theoretical studies

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*Abstract:* It is a common problem that KDD processes may generate a large number of patterns depending on the algorithm used, and its parameters. It is hence impossible for an expert to assess these patterns. This is the case with the well-known APRIORI algorithm. One of the methods used to cope with such an amount of output depends on using association rule interestingness measures. Stating that selecting interesting rules also means using an adapted measure, we present a formal and an experimental study of 20 measures. The experimental studies carried out on 10 data sets lead to an experimental classification of the measures. This study is compared to an analysis of the formal and meaningful properties of the measures. Finally, the properties are used in a multi-criteria decision analysis in order to select amongst the available measures the one or those that best take into account the user's needs. These approaches seem to be complementary and could be useful in solving the problem of a user's choice of measure.

*Keywords:* association rule, interestingness measure, interestingness criteria, measure classification, measure selection.

## Introduction

One of the main objectives of Knowledge Discovery in Databases (KDD) is to produce interesting patterns. This notion of interest highly depends on the user's goals. This user is not assumed to be a data mining expert, but rather an expert in the field being mined. Moreover, it is well known that the interestingness of a pattern is difficult to evaluate objectively. Indeed, this estimation greatly depends on the expert user's interests [1], [2]. Ideally, a

pattern should be *valid, new and comprehensive* [3], but these generic terms cover a large number of situations when examined in a precise context. It is a common problem that data mining algorithms produce a huge amount of output, and that the end user is then unable to analyse it individually. What is more, a large part of this output is uninteresting [4]. Thus, when dealing with pattern selection one has to face two problems: the quantity and the quality of rules. This is particularly true when mining association rules with the well-known algorithms of the APRIORI family, within a support-confidence framework [5], and this is the issue that we will assess.

In this context, different solutions, more or less involving the user [6], can be considered. Visual data mining uses human visual capabilities to explore the data and patterns discovered (e.g. [7], [8], [9], [10], [11]). Human centred approaches emphasize the cooperation between the user and learning algorithms (e.g. [12], [13] [14], [15]).

Finally, interestingness measures can be used in order to filter and/or sort discovered rules (e.g. [16], [2], [17], [18], [19]). Generally, one distinguishes between objective and subjective interestingness measures. Objective measures are said to be *data-driven* and only take into account the data cardinalities. Subjective measures are *user-driven* in the sense that they take into account the user's *a priori* knowledge and goals. For a discussion about subjective aspects of rule interestingness measures, the reader can refer to [20], [21] and [22].

It should be noted that, in practice, both objective and subjective approaches should be used to select interesting rules [23], the objective ones serving as a kind of first filter to select potentially interesting rules, while the subjective ones can be used as a final filter to retain only the truly interesting rules, depending on the applicative context.

We will focus on objective interestingness measures and take into account both user preferences or goals for association rule discovery and the nature of the data being mined. Such rules were defined in [5]: given a typical market-basket (transactional) database  $E$ , the association rule  $A \rightarrow B$  means *if someone buys the set of items A, then he/she probably also buys item B*. It is of importance to make the distinction between the association rule  $A \rightarrow B$ , which focuses on cooccurrence and gives asymmetric meaning to  $A$  and  $B$ , and logical implication  $A \Rightarrow B$  or equivalence  $A \Leftrightarrow B$  [24].

Interestingness measures play an essential role, reducing the number of discovered rules and retaining only the *best ones*, in a post-processing step.

In order to improve the selection of rules, many classical measures have been used, like the Chi-square test for independency, or the correlation coefficient. Due to specific needs, additional measures have been proposed, such as the lift [25], the  $M_{GK}$  measure [26], relative interestingness [27], the entropic intensity of implication [28], the probabilistic discriminant index [29], the maximal participation index [30], or the h-confidence [31]. As a consequence, a large number of measures are available (see for example [32] for an extensive list of classical measures).

Depending on the user's goals, data mining experts may propose the use of an appropriate interestingness measure, but this selection task cannot be done by the expert user, if left on his own. This choice is hard, since rule interestingness measures have many different qualities or flaws [33], since there is no *optimal* measure. One way to solve this problem is to try to find good compromises [34]. A well-known example of such a controversial measure is the support. On the one hand, it is heavily used for filtering purposes in APRIORI algorithms [5], [35], as its antimonotonicity property simplifies the large lattice that has to be explored. On the other hand, it has almost all the flaws a user would like to avoid, such as variability of the value under the independence hypothesis or the value for a logical rule [36]. Finally, one should be very careful when using the support-confidence framework in defining the interestingness of a rule [37], [38]. To bypass this difficulty different works look for highly correlated items, like as in the CORCLASS algorithm [39] and in the algorithms presented in [40].

It is then relevant to study interestingness measures, so that rules are selected according to the user's needs and context [34]. Interestingness measures have to support KDD process through system-human interaction [41], [42]. Many works (for instance [43], [44], [45], [16], [2], [46], [47], [24], [48], [49], [50], [51], [52]) have formally extracted and studied several specificities of various measures, and the importance of objective evaluation criteria of interestingness measures has already been focused on by [36] and [23].

In this chapter, we will assess the issue of selecting an adapted interestingness measure faced with an applicative context and user's aims.

First, we introduce a set of 20 classical measures which seem applicable in an association rule mining context [53]. In the second section, these measures are analyzed through eight formal properties that make sense from an end user's point of view. In order to highlight the wide variety of measures and have a case based overview of their behaviour, the third section focuses on a tool we have developed, HERBS [54], and an empirical classification of the measures is built out of experimental campaigns [55]. This classification is then compared to another clustering of the measures, based on their theoretical studies. Out of theoretical properties, we finally propose a multi-criteria decision aid (MCDA) approach assessing the issue of selecting an measure adapted to the user's context (aims, goals, nature of the data, etc.) [34]. Finally, we conclude and outline some perspectives that are to be studied.

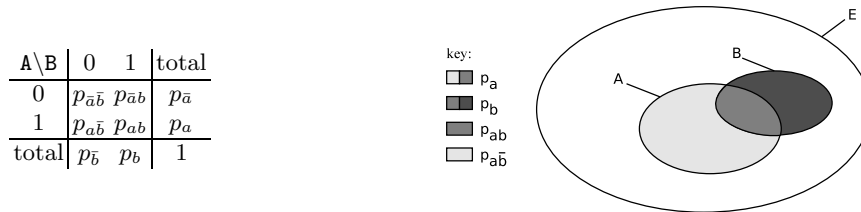
## 1 Interestingness measures

In this section, we present the 20 objective association rules interestingness measures that we studied. These measures are usually defined using the  $2 \times 2$  contingency table presented in figure 1, and is a classical way of measuring association in the case of paired attributes [56], such as in the GUHA method [57], in the 4FT-MINER tool [58] and in the APRIORI algorithm [5].

Given a rule  $A \rightarrow B$ , we note :

- $n = |\mathbf{E}|$  the total number of records in the database  $\mathbf{E}$
- $n_a =$  the number of records satisfying  $A$
- $n_b =$  the number of records satisfying  $B$
- $n_{ab} =$  the number of records satisfying both  $A$  and  $B$  (the examples of the rule)
- $n_{a\bar{b}} = n_a - n_{ab}$  the number of records satisfying  $A$  but not  $B$  (the counter-examples of the rule)

For any  $X$ , we note  $p_x$  instead of  $n_x/n$  when we consider relative frequencies rather than absolute frequencies on the data set  $\mathbf{E}$ . It is clear that, given  $n$ ,  $n_a$  and  $n_b$ , or  $p_a$  and  $p_b$ , knowing one cell of the contingency table in figure 1 is enough to deduce the other ones.



**Fig. 1.** Notations

We restricted the list of measures to decreasing ones, with respect to  $n_{a\bar{b}}$ , all marginal frequencies being fixed. This choice reflects the common assertion that the fewer counter-examples ( $A$  true and  $B$  false) to the rule there are, the higher the interestingness of the rule. Thus some measures like  $\chi^2$ , Pearson's  $r^2$ , Goodman and Smyth's J-measure or Pearl's measure are not considered in this study. The selected measures are listed in table 1, which also includes bibliographical references. Their definition and co-domain, using absolute frequencies, is given in table 2. At first glance, table 2 shows important variations between the formulae. This is due to the fact that measures do not tell the same story. These variations are also noticeable since co-domains are quite different ( $[0, 1]$ ,  $[0, +\infty[$ ,  $]-\infty, 1]$  and others with bounds depending on  $n_a$ ,  $n_b$  and/or  $n_{ab}$ ). For taking into account such variations one may use aggregation operators of valued relations [59] or normalized measures [60].

For a given decreasing monotonic measure  $\mu$  (with respect to  $n_{a\bar{b}}$  margins  $n_a$  and  $n_b$  being fixed), the selection of interesting rules is done by positioning a threshold  $\alpha$  and keeping only the rules satisfying  $\mu(A \rightarrow B) \geq \alpha$ . The value of this threshold  $\alpha$  has to be fixed by the expert, and the same threshold is considered for all the rules extracted during the data mining process. Thus, fixing  $\alpha$  is an important issue [38].

**Table 1.** List of selected measures

	Name	References
BF	Bayes factor	[61]
CENCONF	centred confidence	
CONF	confidence	[5]
CONV	conviction	[62]
ECR	examples and counter-examples rate	
EII	entropic intensity of implication	[28]
IG	information gain	[63]
- IMPIND	implication index	[64]
INTIMP	intensity of implication	[65]
KAPPA	Kappa coefficient	[66]
LAP	Laplace	[67]
LC	least contradiction	[68]
LIFT	Lift	[25]
LOE	Loevinger	[69]
PDI	probabilistic discriminant index	[29]
PS	Piatetsky-Shapiro	[36]
R	Pearson's correlation coefficient	[70]
SEB	Sebag and Schoenauer	[71]
SUP	support	[5]
ZHANG	Zhang	[72]

In our set of measures, we kept the well-known support and confidence: these are the two most frequently used measures in algorithms based on the selection of frequent itemsets for association rule extraction [5], [35].

Many other measures are linear transformations of the confidence, enhancing it, by enabling comparisons with  $p_b$ . This transformation is generally achieved by centering the confidence on  $p_b$ , using different scale coefficients (centered confidence, Piatetsky-Shapiro's measure, Loevinger's measure, Zhang's measure, correlation, implication index, least contradiction). It is also possible to divide the confidence by  $p_b$  (lift).

Other measures, like Sebag and Schoenauer's or the rate of examples and counter-examples, are monotonically increasing transformations of confidence, while the information gain is a monotonically increasing transformation of the lift. Thus, these measures will rank rules in the same order and differ, for example, from their semantic meaning [73].

Some measures focus on counter-examples, like the conviction or the above-cited implication index. This latter measure is the basis of several different probabilistic measures like the probabilistic discriminant index, the intensity of implication, or its entropic version, which takes into account an entropic coefficient, enhancing the discriminant power of the intensity of implication. These last two measures were adapted in order to let them have the desired property of being constant under a null hypothesis (this property is discussed in section 2). For the intensity of implication, the statistical law was ap-

**Table 2.** Association rule quality measures

	Definition	Co-domain
BF	$\frac{n_{ab}n_{\bar{b}}}{n_b n_{a\bar{b}}}$	$[0, +\infty[$
CENCONF	$\frac{nn_{ab}-n_a n_b}{nn_a}$	$[-\frac{n_b}{n}, \frac{n_{\bar{b}}}{n}]$
CONF	$\frac{n_{ab}}{n_a}$	$[0, 1]$
CONV	$\frac{n_a n_{\bar{b}}}{nn_{a\bar{b}}}$	$[\frac{n_{\bar{b}}}{n}, +\infty[$
ECR	$\frac{n_{ab}-n_{a\bar{b}}}{n_{ab}} = 1 - \frac{1}{\frac{n_a}{n_{a\bar{b}}}-1}$	$] -\infty, 1]$
EII	$\{[(1-h_1(\frac{n_{a\bar{b}}}{n}))^2(1-h_2(\frac{n_{a\bar{b}}}{n}))^2]^{1/4} \text{INTIMP}\}^{1/2}$	$[0, 1]$
IG	$\log(\frac{nn_{ab}}{n_a n_b})$	$] -\infty, \log \frac{n}{n_b}]$
-IMPIND	$\frac{n_a n_b - nn_{ab}}{\sqrt{nn_a n_b}}$	$[-\sqrt{\frac{n_a n_b}{nn_a n_b}}, \sqrt{\frac{n_a n_b}{nn_a n_b}}]$
INTIMP	$P[N(0, 1) \geq \text{IMPIND}]$	$[0, 1]$
KAPPA	$2 \frac{nn_{ab}-n_a n_b}{nn_a+n_{\bar{a}}n_b-2n_a n_b}$	$[-2 \frac{n_a n_b}{n_a n_{\bar{b}}+n_{\bar{a}}n_b}, 2 \frac{n_a n_{\bar{b}}}{n_a n_{\bar{b}}+n_{\bar{a}}n_b}]$
LAP	$\frac{n_{ab}+1}{n_a+2}$	$[\frac{1}{n_a+2}, \frac{n_a+1}{n_a+2}]$
LC	$\frac{n_{ab}-n_{a\bar{b}}}{n_b}$	$[-\frac{n_a}{n_b}, \frac{n_a}{n_b}]$
LIFT	$\frac{nn_{ab}}{n_a n_b}$	$[0, \frac{n}{n_b}]$
LOE	$\frac{nn_{ab}-n_a n_b}{n_a n_{\bar{b}}}$	$[-\frac{n_b}{n_{\bar{b}}}, 1]$
PDI	$P[\mathcal{N}(0, 1) > \text{IMPIND}^{CR/\mathcal{B}}]$	$]0, 1[$
PS	$n_{ab} - \frac{n_a n_b}{n}$	$[-\frac{n_a n_b}{n}, \frac{n_a n_{\bar{b}}}{n}]$
R	$\frac{nn_{ab}-n_a n_b}{\sqrt{nn_a n_b n_{\bar{a}} \cdot n_{\bar{b}}}}$	$[-\sqrt{\frac{n_a n_b}{nn_a n_b}}, \sqrt{\frac{n_a n_{\bar{b}}}{nn_a n_b}}]$
SEB	$\frac{n_{ab}}{n_{a\bar{b}}}$	$[0, +\infty[$
SUP	$\frac{n_{ab}}{n}$	$[0, \frac{n_a}{n}]$
ZHANG	$\frac{nn_{ab}-n_a n_b}{\max\{n_a n_{\bar{b}}, n_b n_{a\bar{b}}\}}$	$[-1, 1]$

$\text{IMPIND}^{CR/\mathcal{B}}$  corresponds to  $\text{IMPIND}$ , centred reduced ( $CR$ ) for a rule set  $\mathcal{B}$ .

$$h_1(t) = -(1 - \frac{n-t}{n_a}) \log_2(1 - \frac{n-t}{n_a}) - \frac{n-t}{n_a} \log_2(\frac{n-t}{n_a}) \text{ if } t \in [0, n_a/(2n)]; \text{ else } h_1(t) = 1$$

$$h_2(t) = -(1 - \frac{n-t}{n_{\bar{b}}}) \log_2(1 - \frac{n-t}{n_{\bar{b}}}) - \frac{n-t}{n_{\bar{b}}} \log_2(\frac{n-t}{n_{\bar{b}}}) \text{ if } t \in [0, n_{\bar{b}}/(2n)]; \text{ else } h_2(t) = 1$$

$\mathcal{N}(0, 1)$  stands for the centered and reduced normal repartition function

proximated using the centred and reduced normal distribution function. The entropic intensity of implication was modified, according to the definition of the truncated entropic intensity of implication, TEII, as presented in [74].

The bayesian factor, also called sufficiency in [75] or odd-multiplier by [73], is a kind of odd-ratio, based on the comparison of the odd of A and B on B rather than the odd of A and  $\bar{A}$  on B. It has been thoroughly studied in [76].

Finally, Laplace’s measure is a variant of the confidence, taking the total number of records  $n$  into account.

## 2 Evaluation properties

In this section, we propose a list of eight meaningful properties to evaluate the previous list of measures. We present each property, explaining its interest and the modalities it can take.

Two actors take part in this analysis: the user who is an expert of the data mined, whose problem is to select the *best rules*, and the analyst, a specialist of MCDA and KDD, who tries to help the expert. We call the former  $E_r$  and the latter  $E_a$ .

For some properties, a preference order on the modalities they can take is straightforward. These properties can be considered as criteria by  $E_a$  without the intervention of  $E_r$ , namely  $g_1, g_2, g_3, g_4$  and  $g_7$ , and will be called normative. In addition to these, the properties  $g_5, g_6$  and  $g_8$  need  $E_r$  to express his preferences on the values they can take, and will be called subjective [49].

For normative properties, we note **yes** if the measure has the desired property and **no** otherwise.

Table 3 recalls the semantics and the number of modalities of the 8 properties. The results of the evaluations are summarized in table 4.

Property  $g_1$ : **asymmetric processing of A and B** [23]. Since the head and the body of a rule may have a very different signification, it is desirable to distinguish measures that give different evaluations of rules  $A \rightarrow B$  and  $B \rightarrow A$  from those that do not. We note **no** if the measure is symmetric, **yes** otherwise.

Property  $g_2$ : **decrease with  $n_b$**  [36]. Given  $n_{ab}$ ,  $n_{a\bar{b}}$  and  $n_{\bar{a}b}$ , it is of interest to relate the interestingness of a rule to the size of B. In this situation, if the number of records verifying B (i.e. verifying B but not A) increases, the interestingness of the rule should decrease. We note **yes** if the measure is a decreasing function with  $n_b$ , **no** otherwise.

Property  $g_3$ : **reference situations, independence** [36]. To avoid keeping rules that contain no information, it is necessary to eliminate the  $A \rightarrow B$  rule when A and B are independent, which means that the probability of obtaining B is independent of the fact that A is true or not. A comfortable way of dealing with this is to require that a measure’s value at independence should be constant. We note **yes** if the measure’s value at independence is constant and **no** otherwise.

Property  $g_4$ : **reference situations, logical rule [53]**. Similarly, the second reference situation we consider is related to the value of the measure when there is no counter-example. Depending on the co-domain (see table 2), three cases arise. First, the measure takes a value independent of the marginal frequencies (see table in figure 1) and thus takes a constant and maximal value<sup>4</sup>. A second case is considered when the measure takes an infinite value when  $n_{a\bar{b}}$  is null. Finally, a third and more uncomfortable case arises when the value taken by the measure depends on the marginal frequencies when  $n_{a\bar{b}} = 0$ . It is desirable that the value should be constant or possibly infinite. We note **yes** in the cases of a constant or infinite value, **no** otherwise.

We do not take into account the value for the incompatibility situation. The latter reference situation is obtained when  $A \cap B = \emptyset$ , and expresses the fact that **B** cannot be realized if **A** already is. Our choice is based on the fact that incompatibility is related to the rule  $A \rightarrow \bar{B}$  and not  $A \rightarrow B$ .

Property  $g_5$ : **linearity with  $p_{a\bar{b}}$  around  $0^+$  [48]**. Some users express the desire to have a weak decrease in the neighborhood of a logic rule rather than a fast or even linear decrease (as with confidence or its linear transformations). This reflects the fact that the user may tolerate a few counter-examples without significant loss of interest, but will definitely not tolerate too many of them. However, the opposite choice may be preferred as a convex decrease with  $n_{a\bar{b}}$  around the logic rule increases the sensitivity to a false positive. We hence note **convex** if the measure is convex with  $n_{a\bar{b}}$  near 0, **linear** if it is linear and **concave** if it is concave.

Property  $g_6$ : **sensitivity to  $n$  (total number of records) [24], [48]**. Intuitively, if the rates of presence of **A**,  $A \rightarrow B$ , **B** are constant, it may be interesting to see how the measure reacts to a global extension of the database (with no evolution of rates).

If the measure increases with  $n$  and has a maximum value, then there is a risk that all the evaluations might come close to this maximum. The measure would then lose its discrimination power. The preference of the user might be indifferent to having a measure which is invariant or not with the dilatation of data. We note **desc** (for descriptive measures) if the measure is invariant and **stat** (for statistical ones) if it increases with  $n$ .

Property  $g_7$ : **easiness to fix a threshold [53]**. Even if properties  $g_3$  and  $g_4$  are valid, it is still difficult to decide the best threshold value that separates interesting from uninteresting rules. This property allows us to identify measures whose threshold is more or less difficult to locate. To establish this property, we propose to proceed in the following (and very conventional) way by providing a sense of the strength of the evidence against the null hypothesis, that is, the p-value. Due to the high number of tests, this probability should not be interpreted as a statistical risk, but rather as a control parameter [24]. In some cases, the measure is defined as such a probability. More

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<sup>4</sup> Recall that due to our eligibility criterion, we restrict our study to decreasing measures with respect to  $n_{a\bar{b}}$ , all marginal frequencies being fixed.



generally, we can define such a threshold from one of the three types of models proposed by [77] to establish the law followed by  $n_{a\bar{b}}$  under the hypothesis of link absence. We note **yes** if the measure easily supports such an evaluation, and **no** otherwise.

Property  $g_8$ : **intelligibility** [53]. Intelligibility denotes the ability of the measure to express a comprehensive idea of the interestingness of a rule. We will consider that a measure is intelligible if its semantics is easily understandable by the expert of the data  $E_r$ <sup>5</sup>. We assign the value **yes** to this property if the measure can be expressed in that way, **avg** if the measure can be estimated with common quantities, and **no** if it seems impossible to give any simple concrete explanation of the measure.

**Table 3.** Properties of the measures

Property	Semantics	Modalities
$g_1$	asymmetric processing of A and B	2
$g_2$	decrease with $n_b$	2
$g_3$	reference situations: independence	2
$g_4$	reference situations: logical rule	2
$g_5$	linearity with $n_{a\bar{b}}$ around $0^+$	3
$g_6$	sensitivity to $n$	2
$g_7$	easiness to fix a threshold	2
$g_8$	intelligibility	3

The extension of this list is currently being studied, and in particular discrimination, antimonotonicity, and robustness to noise. Discrimination is quite interesting since it might be related to criteria  $g_6$  (sensitivity to the cardinality of the total space), which generally occurs simultaneously with a loss of discrimination. Antimonotonicity also is an interesting property from the computing point of view, both for APRIORI algorithms and Galois lattice based methods [35]. Robustness to noise has been focused on in [78] and [79].

Finally, different alternatives could be proposed for property  $g_3$  (independence). It could be interesting to replace the independence condition ( $p_{b/a} = p_b$ ) by the equilibrium condition ( $p_{b/a} = 0.5$ ) that corresponds to predictive purposes [80]. More generally, a confidence threshold  $\theta$  ( $p_{b/a} = \theta, p_b < \theta < 1$ ) could be taken into account, especially for targeting purposes [74].

<sup>5</sup> It is obvious that this property is subjective. The evaluations of the measures on this property given hereafter can be commonly accepted. Nevertheless, depending on  $E_r$ , our evaluations could be revised.

**Table 4.** Evaluation matrix

	$g_1$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$g_8$
BF	yes	yes	yes	yes	convex	desc	yes	yes
CENCONF	yes	yes	yes	no	linear	desc	yes	yes
CONF	yes	no	no	yes	linear	desc	yes	yes
CONV	yes	yes	yes	yes	convex	desc	yes	avg
ECR	yes	no	no	yes	concave	desc	yes	avg
TEII	yes	yes	yes	no	concave	stat	no	no
IG	no	yes	yes	no	concave	desc	yes	no
- IMPIND	yes	yes	yes	no	linear	stat	yes	no
INTIMP	yes	yes	yes	no	concave	stat	yes	no
KAPPA	no	yes	yes	no	linear	desc	yes	no
LAP	yes	no	no	no	linear	desc	yes	no
LC	yes	yes	no	no	linear	desc	yes	avg
LIFT	no	yes	yes	no	linear	desc	yes	yes
LOE	yes	yes	yes	yes	linear	desc	yes	avg
PDI	yes	yes	yes	no	concave	stat	yes	no
PS	no	yes	yes	no	linear	stat	yes	avg
R	no	yes	yes	no	linear	desc	yes	avg
SEB	yes	no	no	yes	convex	desc	yes	avg
SUP	no	no	no	no	linear	desc	yes	yes
ZHANG	yes	yes	yes	yes	concave	desc	no	no

### 3 Interestingness measure classifications

Beyond a formal analysis, based on meaningful properties, it is interesting to observe the behavior of the measures on data. We present an experimental classification based on preorder comparisons, these preorders being induced by interestingness measures on rule sets. This classification is carried out using our experimentation tool, HERBS. A formal classification based on the formal properties is proposed using a hierarchical ascendent clustering. Finally, we compare the two classifications.

#### 3.1 An overview of HERBS, an experimentation tool

The aim of HERBS [54], [81] is to analyse rule sets and compare or investigate interestingness measures through concrete experiments. It has been designed as an interactive *post-analysis* tool, and hence data sets, rule sets and interestingness measures are considered as inputs. Various useful experimentation schemes are implemented in HERBS, from simple descriptive statistics about rule sets, to comparative overviews of the evaluation of a rule set by several measures.

We here propose an experimental analysis and comparison of measures, based on their application to 10 pairs of data sets and rule sets. A synthetic

comparison of the rankings of a rule set by the measures is given by computing a preorder agreement coefficient,  $\tau_1$  which is derived from Kendall's  $\tau$  (see [82]). This agreement compares a pair of preorders induced by two measures, and its value is in the range  $[-1; 1]$ . The maximum value is obtained when the two pre-orders are equal, whereas the minimum value is obtained in various cases, and especially for reversed preorders.

From a computational point of view, using such a coefficient can be seen as complex since its evaluation is done in  $\mathcal{O}(\eta^2)$ , where  $\eta$  is the number of rules in the rule set considered, when a correlation analysis can be done in  $\mathcal{O}(\eta)$  (the correlation index between interestingness measures is used in the ARQAT tool [83] for example). Still, from the numerous coefficients presented in [82], the  $\tau_1$  coefficient best suits our needs. What is more, HERBS uses a relational database in order to store the experimental results. Building an index on these values greatly optimizes the computation of this coefficient. Finally, only a slight modification of the formula is required in order to return to more classical agreement coefficients, such as Kendall's  $\tau$  or Spearman's  $\rho$ .

### 3.2 Experimental classification

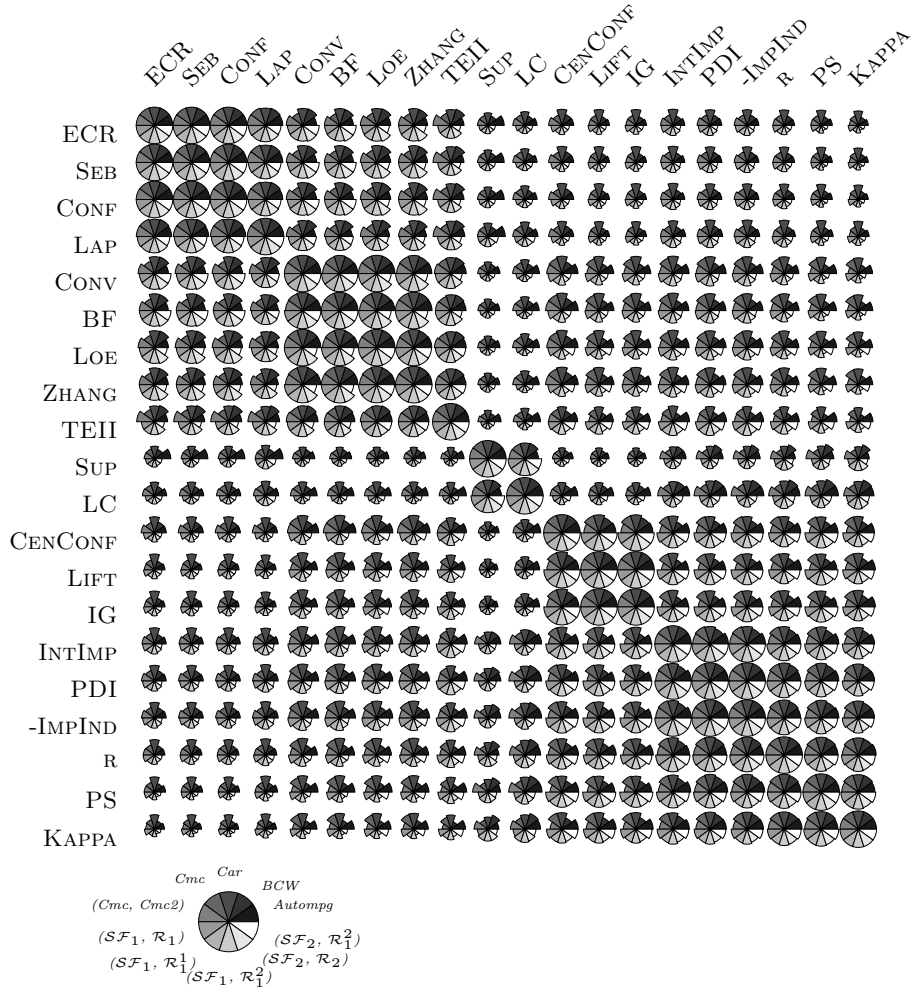
Experiments were carried out on databases retrieved from the UCI Repository (<ftp://ftp.ics.uci.edu/> [84]). When there is no ambiguity, we will refer indifferently to the pair formed by a data set and a rule set, or to the single data set or rule set, using their names in the Repository. We denote by *BCW* the *breast-cancer-wisconsin* database. The parameters of the APRIORI algorithm [85] were fixed experimentally in order to obtain rule sets of an acceptable size in terms of computational cost (see table 5). The great differences in size of the rule sets is related to the number of modalities of the different attributes of the case databases. A particular option was used in order to compute *Cmc*: APRIORI, which usually explores a restricted number of nodes of the lattice formed by the different modalities of the attributes, was forced to explore the entire lattice. *Cmc2* was obtained by filtering *Cmc*, with a minimum lift of 1.2. The *Solarflare* database is divided into two case sets,  $\mathcal{SF}_1$  and  $\mathcal{SF}_2$ , described by the same attributes.  $\mathcal{R}_1$  (resp.  $\mathcal{R}_2$ ) is the rule set coming from  $\mathcal{SF}_1$  (resp.  $\mathcal{SF}_2$ ). We filtered  $\mathcal{R}_1$ , with the method exposed in [86] following the results of [87] in order to keep only rules that are significant from a statistical point of view. Using  $\mathcal{SF}_1$  (resp.  $\mathcal{SF}_2$ ), we obtained the rule set  $\mathcal{R}_1^1$  (resp.  $\mathcal{R}_1^2$ ). The characteristics of the sets are summarized in table 5.

We generated 10 preorder comparison matrices, which are presented in table 6 (the value of  $\tau_1$  is proportional to the radius of the corresponding portion of disc, a radius null corresponding to an agreement of  $-1$ , and a radius of 1 corresponding to an agreement value of 1). The AMADO method [88] was applied to the average matrix of the results in order to reorganize the rows and the columns of this matrix, and highlight the block structures. The results are quite in agreement, and we can make out 3 main groups of measures, and in two of these groups we can distinguish two subgroups (see tables 6 and 7).

**Table 5.** Summary of the different sets used, and APRIORI parameters

name	$n$	$sup_{min}$	$conf_{min}$	$\eta$	name	$n$	$sup_{min}$	$conf_{min}$	$\eta$
<i>Autompg</i>	392	5	50	49	$(\mathcal{SF}_1, \mathcal{R}_1)$	323	20	85	5402
<i>BCW</i>	683	10	70	3095	$(\mathcal{SF}_2, \mathcal{R}_2)$	1066	20	85	6312
<i>Car</i>	1728	5	60	145	$\mathcal{R}_1^1$	$n/a$	$n/a$	$n/a$	4130
<i>Cmc</i>	1473	5	60	2878	$\mathcal{R}_1^2$	$n/a$	$n/a$	$n/a$	2994
<i>Cmc2</i>	$n/a$	$n/a$	$n/a$	766					

**Table 6.** Preorder comparisons of 20 measures on 10 experiments.



The first group consists of {ECR, SEB,CONF, LAP, CONV, BF, LOE, ZHANG, TEII} and can be sub-categorized into two subgroups:  $E_1 = \{ECR, SEB,CONF, LAP\}$  and  $E_2 = \{CONV, BF, LOE, ZHANG, TEII\}$ . The second main group consists of  $E_3 = \{SUP, LC\}$ , behaving very differently from

the previous measures. The third group,  $\{\text{CENCONF}, \text{LIFT}, \text{IG}, \text{INTIMP}, \text{PDI}, \text{-IMPIND}, \text{R}, \text{PS}, \text{KAPPA}\}$ , can be split into two, as was the first one, and leads to the two following subgroups:  $E_4 = \{\text{CENCONF}, \text{LIFT}, \text{IG}\}$  and  $E_5 = \{\text{INTIMP}, \text{PDI}, \text{-IMPIND}, \text{R}, \text{PS}, \text{KAPPA}\}$ .

### 3.3 Formal classification

The formal approach can be synthesized with a  $20 \times 8$  matrix, containing the evaluation of the 20 measures on the 8 properties. We kept only 6 of the properties for the comparison between experimental and formal approaches, as two of them – namely  $g_7$  (easiness to fix a threshold) and  $g_8$  (intelligibility) – do not influence the experimental results at all.

All these properties are bivariate except  $g_5$  which is trivariate. The  $20 \times 6$  matrix formally obtained was re-encoded in a  $20 \times 6$  matrix composed of real values, 0 or 1 in the binary cases, and 0, 0.5 or 1 for  $g_5$ . These values do not represent any judgement on the measures, but only list the properties shared by the different measures.

The typology in 5 classes,  $F_i$ ,  $i = 1 \dots 5$  (see table 7) coming from this matrix is obtained with a hierarchical ascendant clustering, using the average linkage, applied to the Manhattan distance.

### 3.4 Comparison of the two classifications

Table 7 shows that both approaches globally lead to similar clusterings, but some shifts are interesting. The main differences concern  $\{\text{SUP}, \text{LC}\}$  and  $\text{TEII}$ .

The experimental classification leads to two main classes,  $E_1 \cup E_2$  and  $E_4 \cup E_5$ . The coherence between the two classifications is underlined by the fact that apart from the three above-mentioned measures,  $E_1 = F_1 \cup F_2$ ,  $F_3 \subset E_2$  and  $E_4 \cup E_5 \subset F_4 \cup F_5$ .

From a formal point of view,  $\text{SUP}$  and  $\text{LC}$  are quite close, forming class  $F_2$  together with  $\text{LAP}$ . There also is a strong link between the classes  $F_1$  and  $F_2$ . Apart from  $\text{SUP}$  and  $\text{LAP}$ , the measures belonging to these classes are those sharing the property of making reference to indetermination when evaluating the quality of a rule (i.e. measures having a constant value when  $n_{ab} = n_{a\bar{b}} = n_a/2$ , [89], [80]), although this property was not taken into account in our formal classification.

The formal class  $F_5$  is made out of the measures built on the implication index, namely  $\text{-IMPIND}$  itself,  $\text{INTIMP}$  which is derived from the former through the use of the normal distribution, and the two discriminant measures,  $\text{TEII}$  and  $\text{PDI}$ . In our formal approach no distinction can be made between  $\text{INTIMP}$ ,  $\text{TEII}$  and  $\text{PDI}$ , since none of the criteria  $g_1$  to  $g_6$  take into account the discriminating power of the measures. We are currently working on such a criterion. Apart from  $\text{TEII}$ , these measures make up the same experimental class, which also includes  $\text{R}$ ,  $\text{KAPPA}$  and  $\text{PS}$ . The altered behavior of  $\text{TEII}$  is

**Table 7.** Cross-classification of the measures

Formal \ Experimental	Class $E_1$	Class $E_2$	Class $E_3$	Class $E_4$	Class $E_5$
Class $F_1$	CONF, SEB, ECR				
Class $F_2$	LAP		SUP, LC		
Class $F_3$		CONV, BF, LOE, ZHANG			
Class $F_4$				LIFT, IG, CENCONF	R, KAPPA, PS
Class $F_5$		TEII			INTIMP, -IMPIND, PDI

due to the fact that it is derived from INTIMP through the use of an inclusion index. This inclusion index plays a major role in the evaluation of the quality of a rule and thus accounts for the experimental differences. Experimentally, TEII thus shifts to LOE, ZHANG, BF and CONV (class  $E_2$ ).

Formally, LAP shifts to LC and SUP (class  $F_2$ ). A reason for this shift is that although it is really close to SUP in our formal study, LAP can differ from CONF experimentally only for values of  $n_a$  close to 0 (nuggets). The minimum thresholds of the APRIORI algorithms make this impossible, and this can be seen as an algorithmic bias [55].

Property  $g_4$  has an important impact on experimental results. When it is verified, all the logical rules are evaluated with a maximal value, no matter what the conclusion is. BF, CONV, LOE, ZHANG, and ECR, SEB, CONF, *i.e.* the measures for which  $g_4 = \text{yes}$ , make the experimental group  $E_1 \cup E_2$ . Only TEII and LAP, also belonging to these classes, do not share this property.

## 4 A multi-criteria decision approach towards measure selection

In this section, we will analyze and evaluate the measures described earlier and summarized in table 2. This analysis was done by a few MCDA procedures, in particular the TOMASO method for sorting [90], a ranking procedure based on kernels of digraphs [91] and the PROMETHEE method [92]. These three methods have produced very similar results. In this chapter, we focus on the analysis by the PROMETHEE method to obtain a ranking. A formalization of the decision problem is discussed in [93]. This approach has been used in a real context by [94].

### 4.1 A few words on the PROMETHEE method

Its objectives are to build partial and complete rankings on alternatives (in this case, the measures) and to visualize the structure of the problem in a plane called the GAIA plane, similarly to a principal component analysis. The PROMETHEE method requires information about the importance of the criteria (a criteria is a property on which a preference modeling is known) to be given by a set of weights. Several tools allow these weights to be fixed in order to

represent the decision maker’s preferences ( $E_r$  in our context). The first step of the method is to make pairwise comparisons on the measures within each criterion. This means that for small (large) deviations,  $E_r$  will allocate a small (large) preference to the best measure. This is done through the concept of preference functions. Then, each measure is confronted with the other ones in order to define outranking flows. The positive (negative) outranking flow expresses to what degree a measure  $a$  is outranking (outranked by) the others. Finally, partial and complete rankings are generated from these outranking flows. The GAIA plane provides information on the conflicting character of the criteria and on the impact of the weights on the final decision. It is a projection, based on a net flow  $\phi$  derived from the outranking flows, of the measures and the criteria in a common plane. For a more detailed description of this method, the reader can refer to [95], for example.

## 4.2 Analysis of the quality measures

We consider the following two realistic scenarios for the analysis:

**Sc1:** The expert  $E_r$  tolerates *the appearance of a certain number of counter-examples* to a decision rule. In this case, the rejection of a rule is postponed until enough counter-examples are found. The shape of the curve representing the value of the measure versus the number of counter examples should ideally be concave (at least in the neighbourhood of the maximum); the order on the values of criterion  $g_5$  (non-linearity with respect to the number of counter-examples) is therefore **concave**  $\succ$  **linear**  $\succ$  **convex**, where  $\succ$  means “is preferred to”.

**Sc2:** The expert  $E_r$  refuses *the appearance of too many counter-examples* to a decision rule. The rejection of the rule must be done rapidly with respect to the number of counter-examples. The shape of the curve is therefore ideally convex (in the neighbourhood of the maximum at least) and the order on the values of criterion  $g_5$  is **convex**  $\succ$  **linear**  $\succ$  **concave**.

For both scenarios, for criterion  $g_6$  we assume that the expert prefers a measure which increases with  $n$ , the size of the data. Thus, the order on the values of criterion  $g_6$  is **stat**  $\succ$  **desc**. For the other criteria which are assumed to be normative, the expert has no influence on the order of the values.

We start by analysing the problem with equal weights for the criteria to get a first view of the structure of the problem. The total rankings for the two scenarios are given in table 8.

First, we notice that both scenarios reflect the preferences of  $E_r$  on the shape of the curve. We can see that for **Sc1** the two leading measures are INTIMP and PDI which are both concave. Similarly, for **Sc2**, the two leading measures are BF and CONV which are both convex. This first analysis also shows that the linear measure LOE is a very interesting measure as it is well placed in both scenarios. It stands for a good compromise.

Sensitivity analyses on the weights systems show that small changes in the weights affect the rankings. Nevertheless a closer look shows that these

**Table 8.** Total rankings for scenarios **Sc1** and **Sc2**.

Rank:	1	2	3	4	5	6	7
<b>Sc1:</b>	INTIMP, PDI		LOE	BF	CENCONF	CONV	-IMPIND
<b>Sc2:</b>	BF	CONV	LOE	CENCONF	-IMPIND	PS	SEB
Rank:	8	9	10	11	12	13	14
<b>Sc1:</b>	ZHANG, TEII		PS	ECR	LIFT	CONF	IG
<b>Sc2:</b>	LIFT	CONF	INTIMP, PDI		R, LC		ZHANG
Rank:	15	16	17	18	19	20	
<b>Sc1:</b>	R, LC		SEB	KAPPA	SUP	LAP	
<b>Sc2:</b>	TEII	KAPPA	ECR	SUP	IG	LAP	

modifications only occur locally and that the first positions of the rankings remain stable.

Therefore one can say that for an expert  $E_r$  who has no particular opinion on the importance of the different criteria, or who considers that the criteria are equally important, the rankings of table 8 are quite acceptable.

An analysis of the GAIA planes gives us further indications about the measures. Figure 2 shows the GAIA planes for **Sc1** and **Sc2**.

Let us first note that the percentage of cumulated variance for the first two factors represented by the GAIA plane is 60.20%. The information taken from the GAIA plane should therefore be considered as approximative and conclusions be drawn with great care. First we observe that the measures (triangles in the figure) are distributed homogeneously in the plane. Then we can see that the GAIA plane is well covered by the set of criteria (axes with squares in the figure). We conclude that the description of the measures selected by the criteria is discriminant and only slightly redundant.

The GAIA plane furthermore helps to detect independent and conflicting criteria. The decision axis  $\pi$  (axis with a circle) indicates in what direction the best alternatives are situated for a given weights system.

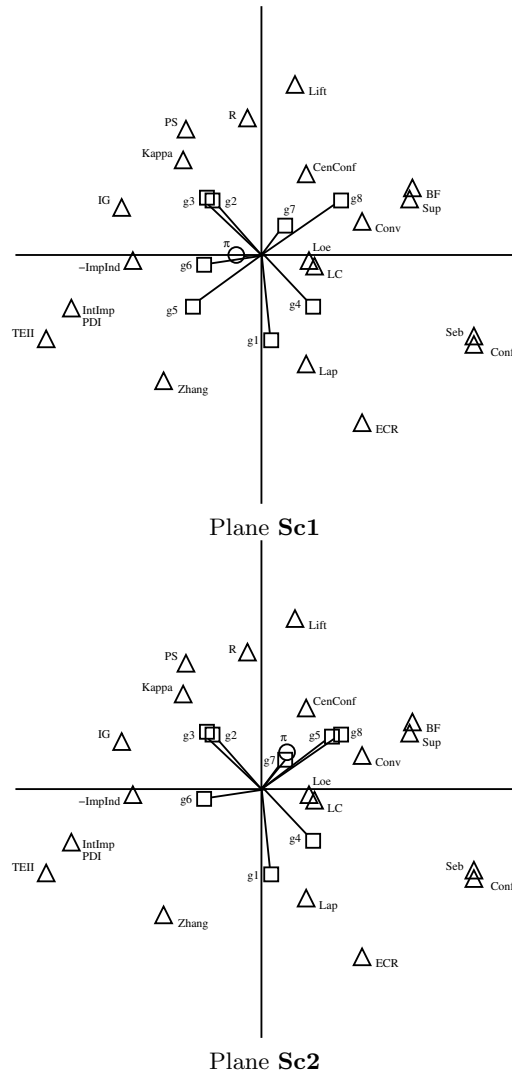
For **Sc1** we can see that several couples of criteria are independent:  $(g_4, g_5)$ ,  $(g_4, g_8)$ ,  $(g_5, g_3)$ ,  $(g_5, g_2)$ ,  $(g_8, g_3)$ ,  $(g_1, g_6)$  and  $(g_8, g_2)$ <sup>6</sup>. We can also observe conflicting criteria. For example  $g_4$  conflicts with  $g_3$  and  $g_2$ ; and criteria  $g_5$  and  $g_6$  conflict with  $g_7$  and  $g_8$ . This type of information gives hints on the behaviour and the structure of the problem. For example, measures which are good for criterion  $g_5$  (concave) will tend to be bad for criterion  $g_8$  (unintelligible).

For **Sc2** similar observations can be made. The major difference lies in criterion  $g_5$  which represents similar preferences to criteria  $g_7$  and  $g_8$  but is conflicting with  $g_6$ .

For **Sc1**  $\pi$  is moderately long and heads in the opposite direction of  $g_7$  and  $g_8$ . This means that measures which allow us to fix the threshold easily and which are easily understandable (and which are quite bad on the remaining criteria) can appear in the leading positions of the ranking only if the relative weights of  $g_7$  and  $g_8$  are very high. However we think that the importance of

<sup>6</sup> If  $g_i$  and  $g_j$  are independent, we write that the couple  $(g_i, g_j)$  is independent.





**Fig. 2.** GAIA planes for **Sc1** and **Sc2**

criterion  $g_3$  (independence hypothesis) should not be neglected compared to a criterion like  $g_8$  (intelligibility). Thus, if the expert is aware of the impact of his weights system on the result, we can suppose that a measure like SUP, exclusively good on  $g_7$  and  $g_8$ , will never appear in the leading positions of the ranking. For **Sc2** the decision axis  $\pi$  is also moderately long. It points in direction of  $g_7$ ,  $g_5$  and  $g_8$ . This partly explains the ranking of table 8.

The positions of the measures in the GAIA plane (for **Sc1** and **Sc2**) show that many alternatives have similar behaviors with respect to weight variations. This is confirmed by their similar profiles in the decision matrix. Thus SEB and CONF, or -IMPIND and PDI are close in the GAIA plane and have similar profiles. These couples of measures will tend to appear in neighbour positions in the rankings. An important comment should be made at this point of the analysis of the GAIA plane. As it represents only a part of the information of the original cloud of points, each observation must be verified in the data or on the basis of other techniques. An erroneous conclusion would be to consider BF and SUP as similar measures due to their proximity in the GAIA plane. In fact, their profiles are very different and, consequently, their behaviour in the case of weight variations will not be similar.

This quite detailed study of the problem shows the utility of an analysis by means of a MCDA tool like PROMETHEE. On the basis of the observations above we can suggest two strategies.

The first strategy involves checking first that the expert  $E_r$  has well understood the meaning of each of the properties. Then, by means of a set of questions, he must express the relative importance of the weights of each criterion. Criteria like  $g_3$ ,  $g_4$  and  $g_7$  will necessarily have high weights to guarantee a certain coherence. Indeed a measure which does not have fixed values at independence and in the situation of a logical rule and, what is more, a threshold which is hard to fix is quite useless in an efficient search for interesting rules. According to the preferences of the expert the relative importance of criteria like  $g_1$  and  $g_8$  can vary. The analysis should be started by using an initial set of weights coherent with these considerations. The stability of the resulting ranking should then be analyzed, especially for the leading positions. If a stable ranking is obtained, the GAIA plane, the value of the net flows and the profile visualization tool allow a finer analysis of the leading measures. The values of the net flows give a hint about the *distance* between two alternatives in the ranking. Two measures with similar values for the net flows can be considered as similar.

The second strategy involves a first step in an exploration of the GAIA plane. This procedure helps the expert to understand the structure of the problem and to detect similar and different measures. Furthermore, the visualization of the criteria in the same plane as the alternatives make it possible to detect the influence of the modification of the weights on the final ranking. This exploratory strategy should be applied with an expert  $E_r$  who has a priori knowledge about certain measures. He will be able to determine a preorder on the importance of the criteria by detecting some well known measures in the GAIA plane. By using this first approximate weights system, the first strategy can be applied. An a posteriori validation can be done by determining the positions of the well known measures in the final ranking.

## 5 Conclusion and perspectives

Association rule quality measures play a major role within a KDD process, but they have a large diversity of properties, which have to be studied both on formal aspects and on real data in order to use a measure adapted to the user's context. In this chapter, we have studied 20 association rule interestingness measures evaluated on 8 properties, and 10 data sets.

The experimental results we present come from a tool we developed, HERBS briefly presented. We were then able to identify 3 main groups of measures in the two approaches, which may be refined in 5 smaller classes. The resulting clusterings are globally in agreement, and the discordancies discussed. The experimental approach seems to be an important addition to the formal approach. Indeed, it first confirmed the validity of the list of formal properties we thought were worth studying. What is more, it has also led to a new reflection on the importance of these properties. For example, requiring that a rule quality measure should have a fixed value for a logical rule has the bias of favouring logical rules with a large conclusion. From the formal study, we proposed a multicriteria decision aid approach illustrating how to help expert users choose an adapted interestingness measure in the context of association rule mining. We present the use of the PROMETHEE decision aid method.

Our approach is a first step to improving the quality of a set of rules that will effectively be presented to the user. Other factors, beyond interestingness measures, can be used. Among them, attribute costs and misclassification costs [23], and cognitive constraints [96].

In addition to the interest of having such a list of properties for a large number of measures, the use of the PROMETHEE method has confirmed the fact that the expert's preferences have some influence on the ordering of the interestingness measures, and that there are similarities between different measures. Moreover, the PROMETHEE method allows us to make a better analysis of the user's preferences (the GAIA plane makes it easy to identify different clusters of criteria and measures).

Our set of criteria covers a large range of the user's preferences, but it is clearly not exhaustive. New criteria could also lead to a better distinction between measures which are similar at the present time. We are confident that some important criteria may also arise from experimental evaluation (such as the discrimination strength and the robustness).

Finally, we would like to point out that even if SUP is poorly rated in both scenarios it is a mandatory measure in algorithms like APRIORI since its antimonotonicity property drives and simplifies the exploration of the lattice of itemsets. In our set of 20 measures, SUP is the only one to have this property.

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