

Operational tools to build a multicriteria territorial risk scale with multiple stakeholders

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Abstract

Evaluating and comparing the threats and vulnerabilities associated with territorial zones according to multiple criteria (industrial activity, population, etc.) can be a time-consuming task and often requires the participation of several stakeholders. Rather than a direct evaluation of these zones, building a risk assessment scale and using it in a formal procedure permits to automate the assessment and therefore to apply it in a repeated way and in large-scale contexts and, provided the chosen procedure and scale are accepted, to make it objective. One of the main difficulties of building such a formal evaluation procedure is to account for the multiple decision makers' preferences. The procedure used in this article, ELECTRE TRI, uses the performances of each territorial zone on multiple criteria, together with preferential parameters from multiple decision makers, to qualitatively assess their associated risk level. We also present operational tools in order to implement such a procedure in practice, and show their use on a detailed example.

Keywords: multicriteria decision aid, group decision making, disaggregation, software

1. Introduction

Assessing risk related to geographical zones requires to take into account possible hazards, and their impacts on possibly different types of assets. Scawthorn [1] cites as assets at risk (in case of earthquakes) social cohesiveness and peace, public confidence, political union, education, mental health (supplementary to physical and non-physical items that have financial value). Subjective judgments may be required

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in order to compare the vulnerability of such different assets and finally obtain a global risk assessment.

This risk assessment activity can thus be seen as a typical task in multicriteria (MC) decision aiding, which enables a formal approach to the aggregation problem when appraising risk. MC decision aiding is a sub-discipline of operations research which explicitly considers multiple criteria in decision-making environments. It is mainly concerned with structuring and solving decision problems where multiple, often conflicting criteria have to be considered.

Other possibilities than using MC methods for taking into account multiple aspects in the risk assessment exist, but they do not attempt to model the preferences of the decision makers in a fine-grained way. E.g., in land use planning, different risk tolerability thresholds may be recommended for hazards threatening hospitals or schools and for hazards threatening office buildings [2, 3]. This article aims at proposing a MC decision aiding approach to help with the subjective part of risk assessment.

Several authors in the risk analysis community have pointed out the potential for MC decision aiding to help in risk assessment and management. Such techniques have been applied to assess risk related to pipelines [4, 5], dredging [6], flood [7], construction projects [8], fire [9], mining-induced hazards [10], technological risk prevention plans [11], land use suitability assessment [12], ... Several of these studies use techniques based on the Electre family of methods that we also use here [8–11]. These methods capture the decision maker's (DM) subjectivity by ways of a set of technical preferential parameters.

A drawback of these studies is that they assume that the decision maker is able to provide the parameters of the models, i.e. they do not propose a way of helping them to determine their values. Furthermore, these studies also assume that a single DM's opinion is to be taken into account.

Other studies have used MC decision aiding methods in a group decision making (GDM) context for risk analysis, but these work by clustering similar opinions together [13] or using mathematical averaging operations such as a weighted sum [e.g. 14–16]. As a result, the outcome may be difficult to interpret, as it does not represent anyone's final opinion but rather some median of the preferences. In contrast, the approach that we propose in this article, aims at obtaining a consensus among all the DMs by letting them solve their disagreements by discussion. This is of course not always possible, but we think it is worth trying to obtain a consensus before resorting to different methods.

We refer to the process of determining the hazards, assets' values, vulnerability (thus possible losses considering the possible hazards), and determination of the geographical zones as risk analysis. In this paper, we assume that such a risk analysis has already been performed upstream. We focus on the subjective part of risk assessment, the one that deals with obtaining an overall risk assessment for a geographical zone by considering the different assets, hazards, and vulnerabilities. Our method does not assume a particular approach for the risk analysis part, but can integrate most results coming from this phase. For example, if the risk analysis uses probabilities to evaluate vulnerabilities, this can be integrated in the risk assessment part. We will come back to this when developing our procedure in section 3.3.

This paper extends the work from Cailloux and Mousseau [17] and Mayag et al. [18] and is structured as follows. Section 2 introduces multicriteria decision aiding

and operational tools to support this process, then presents the relation between a risk assessment problem and a multicriteria sorting problem. In Section 3 we present a specific multicriteria sorting method, ELECTRE TRI, and discuss existing methodologies to elicit an ELECTRE TRI based risk model with stakeholders. We also present the procedure which we recommend for building a multicriteria territorial risk scale with multiple stakeholders. Finally, in Section 4 we detail an illustrative example along with its resolution via operational tools, before concluding in Section 5.

2. Multicriteria decision aiding models for risk assessment

We first present an overview of multicriteria decision aiding as well as operational tools to support it, before motivating the use of such techniques in risk assessment.

2.1. On multicriteria decision aiding

Multicriteria (MC) decision aiding is the activity which provides a decision maker (DM) with a prescription on a set of decision alternatives, when facing multiple, usually conflicting points of view. The DM, who is either a single person or a collegial body, takes the responsibility for the decision act and bears a value system or preferences related to the decision problem, which should be taken into account in the final prescription. The finite set A of decision alternatives represents the potential options on which the DM has to make a decision. The decisions on these alternatives are difficult because multiple conflicting points of view have to be considered. They are represented by a finite set J of criteria indexes.

Usually, three types of problems are put forward in this context [19]:

- the *choice problem* which aims to recommend a subset of alternatives, as restricted as possible, containing the “satisfactory” ones;
- the *sorting problem* which aims to assign each alternative into pre-defined categories or classes;
- the *ranking problem* which aims to order the alternatives by decreasing order of preferences.

Various methodologies have been proposed to support DMs facing a MC decision problem [19–21]. In the following, we first present the outranking school of thought, before switching to the value-based theories.

The main idea behind outranking methods is to compare any two alternatives pairwise on basis of their evaluations on the set of criteria, according to a majority rule. For two alternatives x and y of X , if for the DM there are enough arguments in favor of the statement “ x is at least as good as y ”, then x outranks y ($xS y$) [19]. These arguments are based on differences of evaluations on the various criteria which are compared to discrimination thresholds determined in accordance with the DM’s preferences. Furthermore, a weight is associated with each criterion, which allows to give these local arguments more or less importance in the majority rule. A concordance index then aggregates these partial arguments via a weighted sum to obtain a credibility degree of the outranking.

Three preference situations can be derived from this outranking relation. x and y are considered as indifferent if simultaneously $xS y$ and $yS x$, they are considered as incomparable with respect to the available information if no outranking can be confirmed between them (neither $xS y$ nor $yS x$), and x (resp. y) is strictly preferred to y (resp. x) if $xS y$ and not $yS x$ (resp. $yS x$ and not $xS y$).

As this outranking relation is not necessarily complete or transitive, its exploitation in view of building a decision recommendation is in general quite difficult. Many exploitation procedures have been proposed in the literature to solve the three main types of MC decision problems mentioned above. In Section 3 we present further details on an outranking technique for the sorting problem which is appropriate for risk assessment.

Methods based on multiattribute value theory aim to construct a numerical representation of the DM's preference on the set of alternatives X . More formally, those techniques seek at modeling the preferences of the DM, supposed to be a weak order represented by the binary relation \geq on X , by means of an overall value function $U : X \rightarrow \mathbb{R}$ such that

$$x \geq y \iff U(x) \geq U(y) \quad \forall x, y \in X.$$

The overall value function U can be determined via many different methods, presented for example by von Winterfeldt and Edwards [22, Chapter 8] in the context of an additive value function model. Ideally, such methods should consist in a discussion with the DM in the language of his expertise, and avoid technical questions linked to the model which is used.

Note that the preference relation induced by such an overall value function is necessarily a complete weak order, which means that only two preference situations can occur : either x and y are considered indifferent (if $U(x) = U(y)$) or x (resp. y) is strictly preferred to y (resp. x) if $U(x) > U(y)$ (resp. $U(y) > U(x)$).

Once the overall value function has been properly determined, its exploitation for the decision recommendation is usually straightforward, as all the alternatives have become comparable.

The main differences between these two methodological schools lie in the way the alternatives are compared and in the type of information which is required from the decision maker. Furthermore, outranking methods might be preferable if the evaluations of the alternatives on the criteria are mainly qualitative and if the DM would like to include some impreciseness about his preferences in the model, whereas value-based methods can be favored if the criteria are evaluated mostly on numerical scales and if a compensatory behavior of the DM should be modeled.

2.2. MC decision aiding tools

As mentioned above, there exist several mathematical tools and methodological schools for the resolution of decision problems involving multiple criteria. As a consequence, a large number of software tools have been developed aiming to support this decision aid task [23]. In other scientific fields, as, e.g., statistics or data mining, there exist renowned software platforms which allow to easily compare different analysis methods and to test them on a given data set inside a common framework. Among

the most famous ones, one can cite platforms such as the GNU R statistical system [24] or the Weka suite of machine learning software [25]. Recently, the MC decision aiding field has also given birth to an environment of software tools called *Decision Deck* [26], which allows to facilitate the resolution of MC decision aiding problems for at least three types of users: practitioners who use MC decision aiding techniques to support actual decision makers involved in real world decision problems; teachers who present MC decision aiding algorithms in courses; and researchers who want to test, share and compare algorithms or develop new ones.

Decision Deck is composed of various software-related initiatives, among which one can find diviz [27], a software which eases complex computations from the field of MC decision aiding by simplifying the design, the execution and the sharing of sequences of calculation components (workflows). Among other things it facilitates the construction of these successions of algorithms by allowing the user to combine various calculation elements via an intuitive graphical user interface.

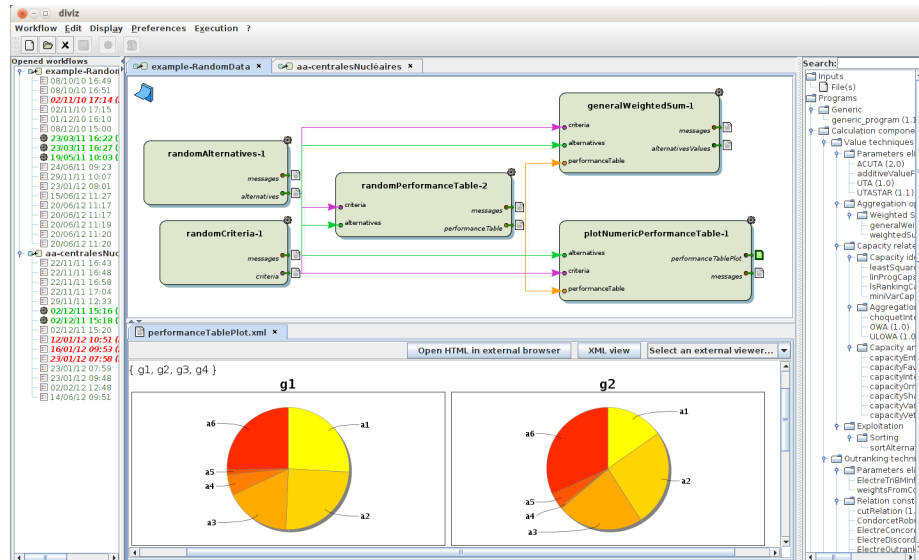


Figure 1: A typical diviz workbench, here showing a workflow and one of its execution results.

Figure 1 shows the diviz workbench. On the left side, a tree presents the list of the opened workflows, along with current and past execution results. The upper-middle panel shows the currently selected workflow, while the lower-middle panel shows the results of one of the components. On the right side, all available programs are organized by themes (e.g. value-based, outranking, ...).

MC decision aiding workflows are built by dragging and dropping the needed calculation components from the right onto the middle panel. These workflows can then be easily executed or shared with other diviz users.

In order to be interoperable, the various algorithmic components available in diviz use a common data standard, XMCD [28], which allows to represent concepts and

data structures coming from the field of MC decision aiding. XMCDAs are written in XML [29], a general-purpose syntax for defining markup languages whose purpose is to aid information systems in sharing structured data, especially via the Internet and to encode documents. XMCDAs are defined via an XML Schema [30], a set of syntax rules and constraints which define its structure.

Each tag of an XMCDAs file describes data related to a decision aid problem. To summarize, these tags can be put in five general categories:

- description of the current decision aiding problem;
- description of the standard MC decision aiding concepts like criteria, alternatives or categories;
- the evaluations of the alternatives on the criteria in the so-called performance table;
- preferences related to criteria, alternatives, attributes or categories (either provided as input by a decision maker or produced as the output of an algorithm);
- output messages from methods or algorithms (log or error messages) and input information for methods or algorithms (parameters).

The various MC decision aiding calculation elements which are available in *diviz* are the XMCDAs web-services [31] proposed by Decision Deck. From a general point of view, a web-service is an application which can be accessed via the Internet and is executed on a remote system. One of the great advantages of such online programs is their availability to anyone at any time and any place and on any computer which is connected to the Internet. From a practical point of view, these web-services propose MC decision aiding algorithms, which, if properly chained (in *diviz* for example), can rebuild MC decision aiding methods. To be interoperable, the inputs and outputs of the XMCDAs web-services are formatted according to the XMCDAs standard.

In *diviz*, once the design of the MC decision aiding workflow is finished, the user can execute it in order to obtain a recommendation for his decision problem. Furthermore, after the execution of the workflow, the outputs of each of the components can be viewed and analyzed by the user. Some of these outputs might represent results of intermediate calculation steps of the workflow, which facilitates the tuning of the parameters of the algorithms.

The history of the past executions is also kept in the software and can at any moment be viewed by the user. More precisely, if a workflow is modified, the former executions' results and their associated workflows are still available. This contributes to the good understanding of the constructed chain of algorithms and to a proper calibration of the preferential parameters of the decision situation.

Next to designing and executing MC decision aiding workflows, *diviz* can also be a convenient tool to compare the outputs of various methods and algorithms on the same input data, or to test the influence of variations in the data on the output recommendation. Indeed, it is easy to connect a data set linked to a specific decision problem to various workflows in a single workspace, each of them representing a different MC decision aiding method, and to compare their outputs. Similarly, variations of the same

data can be connected to multiple copies of the same MC decision aiding technique in order to check the robustness of the output recommendation with respect to different input situations.

The diviz software also enables to export any workflow, with or without the data, as an archive. The latter can then be shared with any other diviz user, who can import it into his software and continue the development of the workflow or execute it on the original data. This allows, in a practical context, for the MC decision aiding treatment to be shared among the various stakeholders of the process.

All in all, these features show that diviz is a very flexible tool and that it can be adapted to various practical decision situations. In particular, as we show in Section 4 on an example, it can be used to support the construction of a risk assessment scale taking into account multiple criteria and DMs. Note that thanks to the workflow sharing feature mentioned above, this example can be downloaded from the diviz website and be tested by any interested reader.

2.3. Defining a qualitative risk assessment scale via a MC sorting model

Most concepts used in a risk assessment problem easily map to concepts used in the MC sorting type of problems. What is called a set of pre-defined categories in those problems represents the set of possible risk levels that the territorial zones must be mapped into. E.g., they can be {High risk, Medium risk, Low risk}. Ideally these risk levels should be defined according to some preventive measures associated with them, to give these categories a precise meaning. The points of view involved in a risk assessment problem, such as the different types of assets whose damages in case of hazard are considered, correspond to the criteria in a MC sorting problem.

Consequently, building a qualitative risk assessment scale amounts to building a MC sorting model.

Each zone being described by a vector of risk factors associated with the points of view involved in the problem, the task at hand consists in assigning these zones to a set of risk categories. This can be done using a MC sorting preference model. This model contains a set of subjective data representing the preferences of the considered DM with regard to, e.g., the relative importance of each of the criteria. These objective and subjective data together with a sorting method allow to aggregate the different points of view to assess the risk level of each considered zone.

These preferential parameters may be elicited in a direct way, but this is often difficult as it requires the DM to understand the fine details of their use in the considered MC sorting method. That is why it has been suggested to deduce the preferential parameters in an inverse way, by asking the DM examples of alternatives, or zones, and the category, or risk level, they would consider appropriate for these.

A supplementary difficulty arises when the evaluation method to be defined involves multiple DMs, as different stakeholders may favor different subjective parameter values. Applying inverse elicitation in a multiple DMs context amounts to ask each DM for a set of examples, which may be conflicting, and deduce preferential parameter values that may be either entirely shared by the DMs, or shared for a part of the parameters, and individual for other values.

In the following section, we present an approach based on an MC sorting procedure taking into account multiple DMs in the elicitation of preferential parameters, and

which can be used to define a qualitative risk assessment scale. The proposed method uses the outranking paradigm presented earlier. This choice is motivated by the facts that in this context of risk assessment, some of the criteria deduced from the points of view will be evaluated on qualitative scales, and that the output risk scale is also ordinal.

3. Electre Tri for risk assessment

The MC sorting method used here is a simplified version of ELECTRE TRI [32–34]. It is appropriate for a risk assessment setting as it only requires ordinally evaluated performances on the different criteria. The version considered here is very close to the version studied by Bouyssou and Marchant [35, 36].

3.1. Sorting procedure

ELECTRE TRI requires, as a definition of the preferences of a DM, criteria importance parameters and category limits separating the categories. The criteria importance parameters include a weight for each of the criteria and a majority threshold that defines when a coalition of criteria is good enough to be decisive. The category limits separate, for each criterion, two consecutive risk levels.

Consider a finite set of territorial zones A , a set of category limits $B = \{b_1, \dots, b_k\}$, and a finite set of criteria indexes J . A criterion g_j ($j \in J$) is a function from $A \cup B$ to \mathbb{R} where $g_j(a)$ denotes the performance of the zone a on the criterion g_j . The zones have to be sorted in k risk levels, c_1, \dots, c_k , ordered by their desirability. c_1 is the worst (i.e. highest) risk level, and c_k is the best (the lowest) one. Each risk level c_h is defined by the performances of its lower frontier, or category limit, b_{h-1} and its upper frontier b_h of B (except the worst risk level c_1 has no lower frontier). The performances are here supposed to be such that a higher value denotes a better performance (i.e. associated with less risks) and the performances on the frontiers are non-decreasing, i.e. $\forall j \in J, 2 \leq h \leq k : g_j(b_{h-1}) \leq g_j(b_h)$.

To sort the zones, ELECTRE TRI uses the concept of outranking relation. The assignment rule used here, known as the pessimistic rule, assigns a zone a to the highest possible risk level c_h such that the zone outranks the category's lower frontier b_{h-1} . A zone a outranks a frontier b_{h-1} if and only if there is a sufficient coalition of criteria supporting the assertion “ a is at least as good as b_{h-1} ”, and no criterion strongly opposes (vetoes) that assertion. To compute this, preferential parameters given by a DM are used. The coalition of criteria in favour of the outranking, $\forall a \in A, 1 \leq h \leq k$, is defined as

$$\sum_{j \in J} w_j C_j(a, b_{h-1}), \quad (1)$$

where w_j is the weight of the criterion g_j , and $C_j(a, b_{h-1}) \in \{0, 1\}$ measures if a is at least as good as b_{h-1} from the point of view of the criterion j or not: $C_j(a, b_{h-1}) = 1 \Leftrightarrow g_j(a) \geq g_j(b_{h-1})$, 0 otherwise. The weights are defined so that they sum to one ($\sum_{j \in J} w_j = 1$). The coalition is compared to a majority threshold $\lambda \in [0.5, 1]$ defined by the decision maker along with the weights. If $\sum_{j \in J} w_j C_j(a, b_{h-1}) < \lambda$, the coalition

is not a sufficient coalition and the zone does not outrank the frontier b_{h-1} and will therefore be assigned in a risk level below c_h .

Even when the coalition is strong enough, a criterion may veto the outranking situation. It happens when $g_j(a) > v_j^{h-1}$. The veto threshold v_j^{h-1} is a value that the DM may define and represents the performance that, if not reached by some zone a , forbids the zone to have a risk label of c_h . To summarize, the zone a outranks the frontier b_{h-1} (and therefore is assigned to at least the category c_h) if and only if $\sum_{j \in J} w_j C_j(a, b_{h-1}) \geq \lambda$ and $\forall j \in J : g_j(a) > v_j^{h-1}$.

In a case involving a single DM, the weights and majority thresholds (defining the sufficient coalitions) and the category limits may be given directly by him. However, this requires that the DM understands how these values will be used. It is moreover a difficult process to directly ask the DM for these parameters. The approach used here supposes that he provides assignment examples which are used to infer the preferential parameters.

The situation is even more complex when several DMs are involved. It is assumed that the order of the categories, the criteria to use, the performances of the zones are consensual. There is no reason however to suppose that all DMs a priori agree on the importance of the criteria or on the frontiers parameters.

3.2. Inference of preferential parameters in a multiple DM context

Recall that the ELECTRE TRI preferential parameter values to elicit are the category limits, the weights, and the vetoes. Previous works aiming to infer preferential parameters for the ELECTRE TRI procedure on the basis of assignment examples usually involve a single DM. Existing approaches suggest to find the entire ELECTRE TRI preference model parameters [37] from assignment examples, or find the importance coefficients only [38], or the categories limits [39], the other parameters being supposedly known. Robust approaches are suggested which compute for each alternative a range of possible categories to which alternatives can be assigned under incomplete determination of the parameters [40–42]. Some tools deal with the problem of non existing preference model solutions which may arise because of an inconsistent set of assignment examples (i.e. assignment examples that do not match ELECTRE TRI) [43, 44]. While the above approaches target a unique DM, Damart et al. [45] propose a method involving a group of DMs that iteratively build, in parallel, individual preference models and a collective preference model representing the group consensus. Table 1 presents a summary of the available tools for indirect preference elicitation related to ELECTRE TRI.

We propose to use the algorithm from Cailloux et al. [46] which proposes a different approach for group decision aiding. Starting from individual assignment examples, it searches for individual preference models that satisfy each DM's examples, with shared category limits. This means that all preference models share the same category limit values and have possibly different weights and majority threshold values. This divide and conquer approach permits to come closer to a consensus, as a part of the model parameters is shared, while still allowing to deal with situations where no unique preference model would be able to represent every DM's assignment examples. The algorithm proposed to infer category limits, named ICL (Infer Category Limits), is a mixed integer linear program. That article also proposes an extension of ICL to infer category limits in case vetoes are used (ICLV algorithm).

Article	input	output
MS98 [37]	i	\mathcal{P}, W (non-linear)
MFN01 [38]	i, \mathcal{P}	W
NM00 [39]	i, W	\mathcal{P}
DMFC02 [42]	i	robust model (\mathcal{P}, W)
MDFGC03 [43]	i^*	how to restore consistency
MDF06 [44]	i^*	how to restore consistency
DDM07 [45]	g, \mathcal{P}	progressive collective model (W)
CMM12 [46]	g	collective model (\mathcal{P})

Table 1: A summary of the main features proposed by other articles and by this one (last row). For each article, the second column indicates the expected input of the main tool proposed in the article, the last one shows its output. i designates assignment examples from a single DM, i^* designates possibly inconsistent assignment examples from a single DM, g is a group of DMs' assignment examples, \mathcal{P} is a set of category limits, W is a set of weights. The computations are based on linear (or mixed integer and linear) programming, except for the first one.

Note that in general, it is unlikely that a unique preference model, with shared category limits and weights, is able to satisfy all assignment examples from every decision makers. Reaching a consensual group preference model usually requires that at least some of the involved decision makers agree to change their minds about some assignment examples. We assume the tool is used in such a context where decision makers may consider changing some of their assignment examples in order to reach a consensual model.

3.3. Building a risk assessment model with multiple stakeholders

This section presents the process we propose to use to build a risk assessment model with multiple stakeholders. We first present the whole process, then indicate some possibilities to transform the output of the risk analysis for our discourse.

3.3.1. Process for building the model

The detailed process is the following.

- Proceed to a risk analysis: obtain a list of hazards and assets that should be considered, as well as a list of possible damage states resulting from hazards occurrence. Evaluate the vulnerability of the assets considering the possible hazards, i.e., the likelihood of damages and amount of losses to the assets, for each possible hazard. A lot of methods have been proposed in the risk literature to proceed to such a risk analysis [47], it is irrelevant to the scope of this paper which one is used. We simply assume that it is possible to transform the output of this risk analysis to a set of evaluations on a set of criteria (this is detailed in the next subsection).
- Determine a list of criteria that matter for the evaluation of the risk pertaining to each zone. Proceed to a geographical cut by grouping together zones that have similar characteristics. Evaluate the zones on each criterion, using data from the risk analysis.

- Obtain from each DM typical zones that correspond to each specific risk levels. These zones are defined by their evaluations on the criteria and will be used as indirect information to elicit the risk assessment model.
- Search for an ELECTRE TRI model without vetoes representing the zone examples with the ICL algorithm.
- If no ELECTRE TRI model without vetoes is able to represent all zone examples, it is possible to allow vetoes to be used in an extension of ICL.
- If still no satisfying ELECTRE TRI model is found, it is possible to check which maximal subsets of zone examples can be represented [43, 44]. The DMs may then be asked individually if they agree to remove or change specific zone examples to restore consistency.
- When shared category limits are found, they should be presented to the DMs for validation. If they disagree, they may propose additional zone examples to further constrain the model and iteratively converge towards satisfactory category limits.
- At any point during the process, DMs may also directly specify some of the preference model parameter values or some veto values. The provided algorithms (ICL and ICLV) are able to take into account such constraints when searching for satisfying preference models.
- At this stage, the DMs agree on a set of category limits but weights of criteria are still possibly distinct for each DM. The approach suggested by Damart et al. [45] may then be used to build a consensus on the weights.

The output of the process are preferential parameters shared among the DMs: a set of criteria weights, category limits and possibly veto thresholds. These can now be used with the ELECTRE TRI model presented in Section 3.1 to sort further zones into the predefined categories in an automated way, i.e. to evaluate these zones on the given risk scale.

One of the important features of the proposed approach is that, as opposed to existing methods, the proposed method applies to a group DM context and does not suppose that part of the preference model is known beforehand.

3.3.2. *From risk analysis to criteria and evaluations*

We give in this section a few possible ways of transforming the output of the risk analysis into an evaluation table suitable for the risk assessment part we are interested in.

- For each asset type, choose one or two indicators of the possible losses such as expected loss or maximal loss, which will constitute the criteria. The expected loss can be computed as the probability of each possible damage resulting from each possible hazard multiplied by the loss occurring in case of that damage occurring. The unit of measure of these losses depend on the asset type and can be qualitative or quantitative: number of human lives lost, number of injuries,

impact on animal population, financial amount of losses due to building damage, impact on social cohesion, ...

- If the analysis focusses on one particular hazard with only one resulting damage state, the likelihood of this hazard occurring can be used as one criterion, whose evaluations can be probabilities or labels indicating the likelihood of this hazard occurring in the zone (or of its consequences reaching the zone). The other criteria indicate the losses for each asset type in case that hazard and damage state occur. A similar approach is used by Salvi et al. [11]. This is the approach we take in the illustrative example that follows.
- When the focus is on a particular hazard, criteria may also be divided into two sets: on the one hand, criteria indicating the susceptibility of that particular hazard to occur, on the other hand measures of the vulnerability of each asset type to that hazard. A similar approach is used by Merad et al. [10]. This is particularly suitable when it is not desired or not possible to represent the likelihood of the hazard occurring using a probability.

Ideally, the risk analysis should be performed knowing that the next phase will consist in applying our proposed method. In such a case, risk analysts may already know which criteria will be used, and may focus on evaluating these values. Back and forth iterations between risk analysis (evaluating the criteria) and risk assessment (merging the evaluations) is also possible, though documenting this precisely is out of the scope of this article (see Tsoukiàs [48] for further details on this subject).

Let us now illustrate this process on an illustrative example based on a hypothetical scenario, and present the use of MC decision aiding tools to support the process.

4. Illustrative example and its implementation

A group of four decision makers would like to develop a scale permitting to evaluate the level of risk of each territorial zone around a given industrial installation related to a possible hazard (e.g. a flood). Each zone is to be determined as belonging to one of the three categories {High risk < Medium risk < Low risk}. Each of these categories is associated with specific preventive measures.

It has been chosen to focus on only one damage state related to that hazard. The probability of being in that damage state has been determined, with some uncertainty, for each zone.

The four members of the decision makers group consider that the following six criteria should be used to evaluate the risk associated with each zone. Each criterion scale is defined so that a higher value is “better”. The first criterion indicate the likelihood of the zone being in a damage state. The other five criteria indicate the amount of losses occurring to each type of asset in case of damage.

- p Probability of the zone to be in a damage state. Due to the uncertainties in the measures, the evaluation uses a 5 points ordinal scale. 4 corresponds to probabilities lower than 10^{-6} , 0 to probabilities higher than 10^{-3} , with intermediate values in between.

- po Proportion of the population which is not injured, evaluated as a percentage.
- ec Likely impact to the ecological system of the zone, a binary assessment.
- bu Amount of damages occurring to the buildings in the zone, evaluated on a 5 points ordinal scale.
- pe Damages to other public or environmental assets, evaluated on a 5 points ordinal scale.
- hi Loss of items having historical value, such as items kept in a museum, evaluated on a 3 points ordinal scale.

We suppose that each decision maker provides 30 examples of zones and their associated risk assessment. These examples could correspond to real zones the decision makers have previously evaluated or fictitious zones defined by their evaluation vectors. A part of the examples is displayed in Table 2. Data used in this illustrative example is available as part of the workflow which can be downloaded from <http://www.diviz.org/workflow.ressArticle.html>.

dm	Zone	p	po	ec	bu	pe	hi	Category
dm1	Zone08	5	91	1	4	4	1	Low
dm1	Zone11	5	93	0	3	3	2	Medium
dm1	Zone12	1	43	1	2	3	3	High
dm1	Zone13	2	91	0	5	4	3	High
dm2	Zone00	3	64	1	4	3	2	Medium
dm2	Zone01	2	84	0	3	5	3	High
dm2	Zone03	2	4	1	3	1	2	High
dm2	Zone05	5	14	1	4	5	3	Low
dm3	Zone00	3	64	1	4	3	2	Medium
dm3	Zone01	2	84	0	3	5	3	Medium
dm3	Zone06	2	9	1	2	2	1	High
dm3	Zone08	5	91	1	4	4	1	Low
dm4	Zone02	3	69	1	5	1	2	Medium
dm4	Zone05	5	14	1	4	5	3	High
dm4	Zone09	5	38	1	3	4	3	High
dm4	Zone10	2	36	0	3	4	1	High

Table 2: Part of the input assignment examples.

Using the assignment examples, the algorithm ICL outlined in Section 3.2 is used to find category limits shared by the decision makers which match their individual assignment examples. The results are provided in Table 3. Category limit values have been rounded up: as the criterion scales only use integers, doing so has no effect on the resulting assignments.

The numerical values found as frontiers by the mathematical program should be interpreted in terms of the scales used for the evaluations of the related point of views.

Frontier	p	po	ec	bu	pe	hi
b_1	3	41	1	3	2	2
b_2	4	84	1	4	4	3

Table 3: Inferred category limits. b_1 separate the categories “High risk” and “Medium risk” and b_2 separate the categories “Medium risk” and “Low risk”.

For example, the evaluation of the frontier b_2 on the point of view of the probability of damage criterion (p), as displayed in Table 3, is 4. This means that an evaluation of 4 on the point of view of the probability of damage for a given zone counts as an argument in favour of that zone to be assigned to a category better than b_2 , i.e. the category “Low risk”. If the zone has an evaluation of at least 3, the evaluation of b_1 on the probability of damage criterion, but does not reach 4, hence, an evaluation of 3, this point of view argues in favour of that zone being assigned to the risk level “Medium risk”. If the evaluation is less than 3, thus is 0 to 2, the risk level recommended by this point of view is “High risk”.

Hence, Table 3 shows a possible set of frontier evaluations that may be shared by all DMs such that when used in an ELECTRE TRI model it is possible to reproduce their zone examples, provided adequate weights are used. At this stage the DMs do not share the weight values yet. Table 4 shows for each DM a set of weights matching the assignment examples with the common category limits. Note that these weights are not the only ones that reproduce all assignment examples with the frontier values shown in Table 3.

DM	p	po	ec	bu	pe	hi	λ
dm1	0.348	0.05	0.204	0.05	0.254	0.094	0.649
dm2	0.223	0.05	0.273	0.05	0.13	0.273	0.774
dm3	0.144	0.237	0.237	0.237	0.05	0.094	0.572
dm4	0.244	0.556	0.05	0.05	0.05	0.05	0.804

Table 4: A set of weights, as found by the ICL program, matching zone examples of each decision maker when used together with the inferred category limits.

However, when presented the category limits, the group disagrees with one of the values, saying that the category limit b_2 on the criterion bu must have a value of 5, thus, that a zone should have an evaluation of 5 according to the point of view of the building vulnerability for this point of view to argue in favour of the zone to be in the “Low risk” category. Furthermore, the group agrees that a performance of only 1 on that criterion should forbid access from that zone to the best category (“Low risk”), whatever the other performances on the other criteria. This can be modelled with a veto. The program ICLV is then used to search for preference models matching all assignment examples and having values $v_{bu}^2 = 1$ and $g_{bu}(b_2) = 5$. The new category limit values are displayed in Table 5, and a set of weights compatible with the zone examples using these frontiers is presented in Table 6.

Once shared category limits have been found, and supposing that all DMs agree

Frontier	p	po	ec	bu	pe	hi
b_1	3	41	1	3	2	2
b_2	4	82	1	5	4	3
v^1	-	-	-	-	-	-
v^2	-	-	-	1	-	-

Table 5: Inferred category limits on the second run. b_1 separates the categories “High risk” and “Medium risk” and b_2 separates the categories “Medium risk” and “Low risk”.

DM	p	po	ec	bu	pe	hi	λ
dm1	0.365	0.05	0.221	0.05	0.221	0.094	0.633
dm2	0.144	0.05	0.512	0.05	0.05	0.194	0.854
dm3	0.144	0.237	0.237	0.237	0.05	0.094	0.572
dm4	0.556	0.244	0.05	0.05	0.05	0.05	0.804

Table 6: A set of weights, as found by the ICLV program, matching zone examples of each decision maker when used together with the inferred category limits.

with the shared frontier values, the approach suggested by Damart et al. [45] may then be used to iteratively build consensual weight values among the group of decision makers. We suppose here that the output of this procedure generates the weights of Table 7, which reconstitute 96 assignment examples from the decision makers on the 120 provided when used with the frontier values and vetoes displayed in Table 5.

DM	p	po	ec	bu	pe	hi	λ
all	0.18	0.36	0.18	0.18	0.05	0.05	0.68

Table 7: A set of weights matching 96 of the zone examples of the total 120 examples when used together with the inferred category limits.

Suppose now that the four decision makers are asked to evaluate the risk associated with six real zones (ZoneA to ZoneF from Table 8). As they have gone through the whole process of eliciting preferential parameters shared by their group, they can apply the ELECTRE TRI assignment rules in order to evaluate these zones automatically. The output evaluation is given in the last column of Table 8.

Let us now show how this illustrative example can be implemented in a decision aiding tool like diviz. Figure 2 represents the workflow used to support the elicitation process as well as the assignment of the real zones. The large rounded boxes represent the calculation modules whereas the smaller rectangles represent various files related to the data from the illustrative example written according to the XMCD format. The workflow contains two instances of the *ElectreTri1GroupDisaggregationSharedProfiles* module (1 and 2) for the elicitation of the shared profiles during the two attempts of the illustrative example. The output of the first module is not reused, as the decision makers were not satisfied with the shared profiles. The module *criteriaDescriptiveS-*

Zone	p	po	ec	bu	pe	hi	ELECTRE TRI risk assessment
ZoneA	5	91	1	4	4	1	Medium
ZoneB	5	93	0	3	3	2	High
ZoneC	1	43	1	2	3	3	Low
ZoneD	2	91	0	5	4	3	High
ZoneE	3	64	1	4	3	2	High
ZoneF	2	84	0	3	5	3	High

Table 8: Real zones to be evaluated according to their risk level, and the ELECTRE TRI assignment.

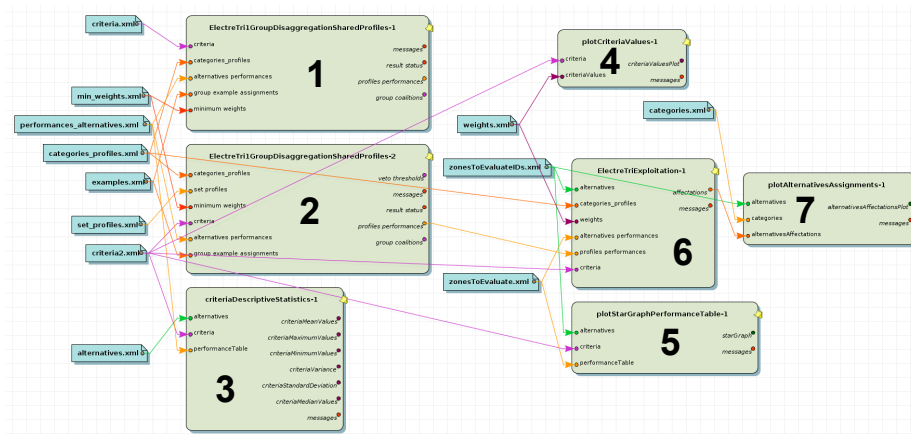


Figure 2: The workflow implementing the illustrative example in diviz.

statistics (3) is used by the decision makers during the elicitation phase to understand the decisions they are facing by computing elementary descriptive statistics on the assignment examples that they are proposing (for each criterion, the mean value, the standard deviation, the maximum and the minimum values are given).

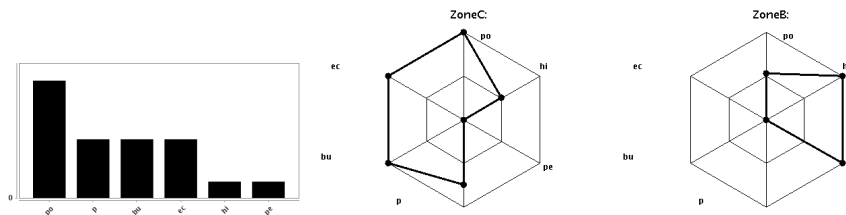


Figure 3: Bar plot of the consensual weights of the criteria and star graph of zone C (resp. B) assigned to category “Low” (resp. “High”) according to the ELECTRE TRI assignment rules.

Once the output of the second elicitation module (2) is validated, the procedure

suggested by Damart et al. [45] to obtain a consensus on the weights is used (without diviz). The resulting weights are plotted via the *plotCriteriaValues* (4) module, whose output is represented in the left part of Figure 3.

The six real zones which are to be evaluated according to their risk are represented graphically via the *plotStargraphPerformanceTable* (5) module. Its output for zone C (resp. B) is represented in the middle (resp. right part) of Figure 3. The output of the profiles elicitation module, together with the consensual weights is then used by the *ElectreTriExploitation* (6) module, which assigns these six real zones to risk levels. Finally, the *plotAlternativesAssignment* module (7) plots these assignments as shown in Figure 4.

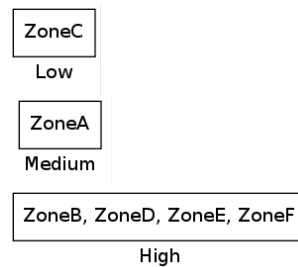


Figure 4: Automated assignments of the 6 zones

The workflow presented here can be downloaded from the website of diviz via the following url: <http://www.diviz.org/workflow.ressArticle.html>. The interested reader can then import this workflow in his diviz tool, and reproduce the calculations from the illustrative example.

5. Conclusion

Risk assessment and management may involve evaluating risk affecting different types of assets, such as human and financial assets. Obtaining a global risk assessment therefore requires to aggregate these different dimensions. We have shown in this article that this task can be achieved by using techniques from multicriteria decision aiding, which aim to reach a consensus among the multiple stakeholders, while evaluating the zones on multiple and potentially conflicting dimensions.

The suggested method permits to generate a meaningful risk scale taking into account multiple stakeholders' judgements. Furthermore, the operational tools presented in this article can be easily used in real-world applications to support the construction of consensual preferences.

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