The RuBy Method for the Recommendation of a Best Choice From a Bipolar Valued Outranking Relation

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Abstract. This short note briefly presents a new and original way to solve the problem of a "best choice" recommendation in a multiple criteria decision aid framework. In particular it discusses how such a "best choice" can be constructed from a binary valued outranking relation defined on a finite set X of potential decision alternatives. The discussion is based on five natural principles.

1 Introduction

The goal of this extended abstract is to discuss how a "best choice"[3] recommendation may be rationally constructed from a binary valued outranking relation defined on a finite set X of potential decision alternatives. Such an outranking relation expresses the likelyhood of a global pairwise preference situation between the alternatives which combines an "at least as good" statement with the absence of any local veto. This decision aid problem is generally non trivial. In practise, most outranking relations result from a multiple criteria preference aggregation involving a majority concordance principle. In general such an aggregation doesn't produce a complete or transitive relation.

From a pragmatic point of view, the BC problematics is turned towards the selection of a unique ultimate "best" alternative. In practise, this kind of decision aid consists in the elicitation of a subset of "good" alternatives which is as restricted as possible. It is meant to help the decision maker to get as close as possible to the selection of a unique "best" alternative. In case this recommendation consists of several candidates, the decision aid process may be restarted with new and more detailed information in order to help selecting the final "best" alternative.

Apart from the European multiple criteria decision aid community [Roy85], this specific BC problematics has attracted quite low attention by the Operational Research field. Seminal work on it goes back to the first articles of Roy on the Electre I methods [Roy68,Roy69]. After Kitainik [Kit93], interest in solving the BC problem differently from the classical optimisation paradigm has reappeared. The recent work of Bisdorff and Roubens on valued kernels [BR96] has resulted in new attempts to solve the BC problem directly from the valued outranking graph. After first positive results [Bis00], methodological difficulties appeared when applying the outranking kernel concept to highly non transitive and partial outranking relations.

In this short note we therefore propose to present the major ideas of a new proposal to the BC problem and to revisit the logical and pragmatic foundations of this problematics. The objective is to propose a new and innovative decision aid methodology in the tradition of the pioneering work of Roy and Bouyssou [RB93].

[3] "best choice" will be written BC in the sequel.

2 Some fundamental concepts

Our starting point is a valued outranking digraph, denoted $\widetilde{G}^{\mathcal{L}}(X, \widetilde{\mathrm{S}})$, where X is a finite set of decision alternatives and $\widetilde{\mathrm{S}} : X \times X \to \mathcal{L}$ is a bipolar valued characterisation of an outranking relation on X taking its values in a bipolar evaluation domain $\mathcal{L}$.

Commonly $\mathcal{L}$ consists of the rational unit interval expressing the more or less credibility or robustness of an outranking statement. Throughout this paper we shall however suppose, except if stated otherwise, that $\mathcal{L} = \{-m, \ldots, 0, \ldots, +m\}$ is a finite ordinal scale with $2m+1$ $(m \geq 1)$ values expressing a degree of likelyhood or robustness. If x and y are two alternatives of X, $\widetilde{\mathrm{S}}(x,y) = m$ signifies that the assertion "x outranks y" is *certainly true*; $\widetilde{\mathrm{S}}(x,y) > 0$ signifies that the assertion "x outranks y" is *more true than false*; $\widetilde{\mathrm{S}}(x,y) = 0$ signifies that the assertion "x outranks y" is *logically undetermined*, i.e. *neither true nor false*; $\widetilde{\mathrm{S}}(x,y) < 0$ signifies that the assertion "x outranks y" is *more false than true*; $\widetilde{\mathrm{S}}(x,y) = -m$ signifies that the assertion "x outranks y" is *certainly false*.

To be short we say that "x outranks y" is *$\mathcal{L}$-true* (respectively *$\mathcal{L}$-false*) if $\widetilde{\mathrm{S}}(x,y) > 0$ (respectively $\widetilde{\mathrm{S}}(x,y) < 0$).

A non empty subset Y of X is called a *choice* in $\widetilde{G}^{\mathcal{L}}$. Such a choice Y is said to be *$\mathcal{L}$-outranking* if and only if either, $Y = X$, or $x \notin Y \Rightarrow \exists y \in Y : \widetilde{\mathrm{S}}(y,x) > 0$. Similarly, a choice Y is said to be *$\mathcal{L}$-outranked* if and only if either $Y = X$, or $x \notin Y \Rightarrow \exists y \in Y : \widetilde{\mathrm{S}}(x,y) > 0$.

A choice Y is said to be *$\mathcal{L}$-independent* if and only if either, Y is a singleton, or $\forall x, y \in Y : \widetilde{\mathrm{S}}(x,y) < 0$. One should notice here that the concept of independence is not based on the negation of the $\mathcal{L}$-true outrankings. Such a negation would also include the couples of alternatives (x,y) for which $\widetilde{\mathrm{S}}(x,y) = 0$ holds.

An *$\mathcal{L}$-outranking* (*$\mathcal{L}$-outranked*) *kernel* is an $\mathcal{L}$-outranking ($\mathcal{L}$-outranked) and $\mathcal{L}$-independent choice.

The goal of our research is to determine a choice Y of X which can be used as a BC recommendation.

3 New foundations for the BC problematics

It is shown in [BRM05] that classical approaches to the BC problem present flaws and weaknesses. For example, the optimisation problem requires that any two alternatives are comparable. The Electre IS method [RB93] requires modifications of the original outranking digraph in order to present a single BC to the decision maker. The concept of $\mathcal{L}$-outranking kernel is also insufficient for the BC problematics, as it may not exist in certain digraphs or be only a subset of possible interesting recommendations.

Therefore we estimate that a new vision of this problem must be adopted. We define a new set of fundamental principles for the BC problematics. The two classical principles defined by Roy [Roy85] are still be present in this set, but are completed by 3 other *natural* ones.

A BC recommendation is a set of alternatives which will be used for a future proposal of a unique best alternative. This definition shows an important characteristic of any BC procedure. It should be interactive and tend towards the proposal of a unique best alternative. This observation is concordant with Roy's definition of the BC problem (see [Roy85]) where this implicit objective is emphasised.

A BC recommendation Y should therefore verify these 5 principles

- $\mathcal{B}_1$ Each alternative which in not selected must be considered as worse as at least one alternative of Y;
- $\mathcal{B}_2$ The subset of retained alternatives Y must be as small as possible;

– $\mathcal{B}_3$ The subset of retained alternatives should not be simultaneously a "best" and a "worst" choice (effective outranking);
– $\mathcal{B}_4$ The BC recommendation cannot contain another smaller BC recommendation (BC-stability);
– $\mathcal{B}_5$ The BC recommendation must be robust (with respect to impreciseness in the data) (robustness).

The interested reader can refer to an extended version [BRM05] of this short note where we detail each of these principles and their consequences. Furthermore we justify the choice of these principle in view of the BC problematics.

4 Solving the BC problematics

As shown in [BRM05], the problems linked to classical approaches of the BC problematics, lead us to define a particular choice, namely the *hyper-kernel*. A choice Y in $\widetilde{G}^{\mathcal{L}}$ is said to be *$\mathcal{L}$-hyper-independent* if it consists of disjoint cordless $\mathcal{L}$-circuits C^p of odd order $(p = 1, 3, \ldots)$[4]. Consequently an $\mathcal{L}$-outranking ($\mathcal{L}$-outranked) *hyper-kernel* is an $\mathcal{L}$-hyper-independent $\mathcal{L}$-outranking ($\mathcal{L}$-outranked) choice. It is possible to show that these hyper-kernels verify the five principles introduced in Section 3. A *pre-hyper-kernel* is a hyper-kernel for which at least one cordless $\mathcal{L}$-circuits C^p of odd order is in an undetermined situation. This means that it is impossible to determine if it belongs or not to the choice. Nevertheless, a deeper analysis on the choice may exclude it or include it for good.

We now present the algorithm for the resolution of the BC problematics:

1. Detection of the odd cordless $\mathcal{L}$-circuits of $\widetilde{G}^{\mathcal{L}}$.
2. Search for the $\mathcal{L}$-outranking ($\mathcal{L}$-outranked) (pre-)hyper-kernels ($\mathcal{B}_1$, $\mathcal{B}_2$ and $\mathcal{B}_4$).
3. Determination of the robust effective $\mathcal{L}$-outranking (pre-)hyper-kernels ($\mathcal{B}_3$ and $\mathcal{B}_5$).
4. Ranking of the robust effective $\mathcal{L}$-outranking (pre-)hyper-kernels according to decreasing *logical determination*.

The BC recommendation is then given by the $\mathcal{L}$-outranking hyper-kernel(s) with the highest degree of logical determination.

5 Concluding remarks

This short note describes the very basic ideas of the RUBY procedure for the determination of a BC in a valued outranking digraph. For further details the interested reader should refer to [BRM05]. The procedure is based on five natural principles and introduces the concept of hyper-kernel of a digraph in order to build a BC recommendation.

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