Practical work: traffic and queueing using a Java simulator

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The objective of this practical session is to see the behavior of a pure waiting queue and of loss systems, in order to better understand Erlang B and C laws, and Engset law.

To that extend, we will use a Java applet to simulate telephone calls for various configurations and for various values of the parameters (call arriving rate, mean call duration, number of subscribers, of servers, duration of the observation period). We will also compare the experimental values obtained by simulation to the theoretical values given by the traffic laws.

Connection

The simulator can be found by clicking on **Vistad** from the url
`http://formations.telecom-bretagne.eu/ressources/rsm/`.

1 Presentation of the simulator

You can choose among three different applets :
- *Traffic without blocking* : we have here a queueing system with an infinite queue length. If a new call arrives while all servers are busy, the call waits until a server is freed to serve it.
- *Loss systems : Erlang law* : we consider a system with no waiting room. At the arrival of a call, either there is an available server that immediately treats the call or the call is lost. We assume that calls arrive according to a Poisson process with a constant parameter λ (independently of the number of calls that are being treated).
- *Loss systems : Engset law* : this is also a system with no waiting room, but here the call arrival process is different from the previous system. We consider here a finite number of subscribers ; each subscriber that is not calling has a Poisson arrival call process with parameter λ_U. Therefore, if N_C subscribers out of the N subscribers are calling, the arrival rate of a new call is $(N - N_C)\lambda_U$.

2 Loss systems : Erlang law

- You can choose in the *simulation parameters* window the values of all parameters that define the process, and the law of the arrival process and of the service time (call duration)

Question 1 Fix the values of the parameters such that the theoretical incoming trafic equal 10 Erlang. What values did you choose ?

Question 2 For a system with 15 servers, run a simulation with duration 120 minutes. *NB :* notice that the simulation speed can be changed using the cursor at the lower left-hand corner of the simulation window.

1. What are the minimum and maximum values taken by the offered traffic over time ?
2. Can the offered traffic be higher than the number of servers ? If so, give a date when it occurs, and the value of the offered traffic at that date.

Question 3 To what events do an increase and a decrease in the offered traffic curve correspond ?

Question 4 Knowing the curve of the offered traffic, can you deduce the curve of the carried traffic by taking the minimum with the number of servers (i.e. do we have $carried \quad traffic = \min(offered \quad traffic, Nb\_servers)$) ? Why ?

Question 5

1. What do the virtual servers represent ?
2. Is it possible to have at the same time some free servers and some busy virtual servers ?

Question 6 We know focus on the three curves : offered traffic, carried traffic, blocked traffic.

1. In which configurations do call losses occur ?
2. What is the relation that links those three curves ?

Question 7 Modify (substantially) the values of the arrival rate and of the mean service rate, such that the theoretical offered traffic still equals 10 Erlang. Run the simulation and check the three curves. What is the difference with the curve of the previous simulation ?

Question 8 With the values that are authorized by the applet, what values of the arrival rate and the mean service rate should give the measured results closest to the theoretical values ?

Question 9 Run another simulation, but changing the date of measurement start. Why was this option introduced ?

Question 10 Run a simulation long enough in order for the measured values to be quite stable. Does the measured value of call loss (ratio *number of rejected calls/total number of calls*) correspond to the theoretical value ?

Run the same simulation, but changing the service law. Compare the measured and the theoretical values when the call duration is Pareto-distributed or is constant.

Question 11 Select again an exponential service law, but change the arrival law, keeping the values of the other parameters unchanged. What measured values do you obtain for the two types of process available in the simulator ? Do those values correspond to the theoretical results ?

3 Loss systems : Engset law

We now work with the *Loss systems : Engset law* simulator.

Question 12 Why do we now have some "customers" icons ?

Question 13 Run a simulation with 10 customers and 14 servers. Are there any blocked calls ?

Question 14 Why can't we see the curves of the offered traffic and of the blocked traffic as with the previous simulator ?

Question 15 For a system with 4 servers, generate an offered traffic of 10 Erlang and observe the theoretical blocking probability (without running the simulation) when there are 5 customers, 10 customers, 15 customers, 20 customers.

Compare to the value given by the Erlang B law.

4 Traffic without blocking

Question 16 Change the values of the parameters. In what conditions does the message "Non-stable simulation" appear ? Why don't we have the theoretical value of the carried traffic ?

Question 17 Run a simulation with 10 servers and an offered traffic of 9.5 Erlang, and observe the histograms and the cumulative functions of the waiting and dwell durations. *Among users who are not served immediately*, what is the proportion of users whose waiting time exceeds twice the mean service duration ? Compare the measured value to the theoretical value.

5 Statistical validity of simulation results

Question 18 Run a simulation with the "Loss systems : Erlang law" applet with exponentially distributed service times of mean 180 seconds, and observe the *service time* measure during the simulation. This measure corresponds to the mean service time among all calls (real and virtual), which are independent and identically distributed variables.

Does the final measured value equal the theoretical value ? How many samples were used to obtain this value ?

We will now use some theoretical results of probability in order to control the estimating error that is made by the simulator because of a too small sample set. The results we use come from the *central limit theorem* :

Theorem 1 *Consider a series* $(X_n)_{n\in\mathbb{N}}$ *of independent and identically distributed random variables with finite mean* m *and variance* σ^2*. Then the empirical mean* $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ *is such that* $\frac{\sqrt{n}}{\sigma}(\bar{X}_n - m)$ *converges in law to a normal distribution* $\mathcal{N}(0,1)$ *with mean* 0 *and variance* 1*.*

Knowing that a random variable with law $\mathcal{N}(0,1)$ belongs to the interval $[-1,1]$ with probability 0.95 and to interval $[-1.96, 1.96]$ with probability 0.99, we can deduce the following application :
For sufficiently large n, the confidence intervals for the estimation of the mean m are :

$$\begin{aligned} 95\% \text{ confidence interval} &: [\bar{X}_n - \frac{1.96\sigma}{\sqrt{n}}, \bar{X}_n + \frac{1.96\sigma}{\sqrt{n}}] \\ 99\% \text{ confidence interval} &: [\bar{X}_n - \frac{2.58\sigma}{\sqrt{n}}, \bar{X}_n + \frac{2.58\sigma}{\sqrt{n}}] \end{aligned}$$

Question 19 Calculate the variance for an exponentially distributed variable with parameter μ, and the number of calls that the simulator should take into account for the error to be less than 2 seconds with probability 95% (resp. 99%). How many more samples do we need to divide the error by 2 ?